front, in agreement with the results of simulation of the problem at $M \gtrsim M_{cr}$ [3]. Such a front structure is similar to the known "isomagnetic jump" of magnetohydrodynamics for waves under conditions of low conductivity [6], where the temperature and the velocity experience a jump, whereas the front of the magnetic field is "smeared out." Using the concept of the toppling of the wave we can explain also the formation of an "advance" layer of perturbed plasma ahead of the jump, as a result of the "spilling over" of the front at $M \gtrsim M_{cr}$, when the turbulent dissipation is insufficient to compensate the nonlinear twisting [2]. Violation of the single-stream nature of the ion motion and the appearance of a viscous mechanism should change the rate of growth of H, as is indeed observed in the experiment. When the Mach number is increased ($M \sim 4 - 5$) the region of the "pedestal" broadens, and the region of the abrupt jump disappears. The resultant relatively smooth n(t) profile (Fig. 3b) apparently corresponds to a "steady" state of the front after its toppling. It is typical that in this case the rate of growth of the magnetic field assumes over the entire front ($\Delta \to Mc/\Omega_0$, where Ω_0 is the ion plasma frequency) approximately the same value as it had in the "pedestal" region.

The measurements of the plasma conductivity and the investigation of the laws governing the electric and magnetic fluctuations in the wave front agree with the foregoing picture of the realignment of the front structure.

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TWO-PHOTON DECAY OF 2s LEVEL OF HYDROGEN

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As is well known, the decay of the metastable 2s state of a hydrogenlike atom proceeds essentially as a result of emission of two photons, with the atom going over into the 1s state. Much progress was made recently in the experimental study of this phenomenon [1]. The spectrum and the lifetime of the transition are calculated theoretically by numerical summation of series, and therefore only several points of the spectrum are known [2]. The purpose of the present note is to present a complete analytic solution of this problem by using an explicit expression for the Green's function of the electron in the Coulomb field in the coordinate representation.

The probability of emission of two photons with frequencies ω_1 and ω_2 have in the non-relativistic limit and the dipole approximation the form * [3]

$$dW = \frac{8a^2}{9\pi}\left(\frac{4\pi}{3}\right)^2 \sum_{M_1 M_2} \Big| <1s \,|\, r Y_1^{M_2}\left(\frac{\mathbf{r}}{r}\right) G_{E_{2s}-\omega_1}(\mathbf{r},\mathbf{r}')$$

$$r' Y_1^{M_1 *}\left(\frac{\mathbf{r}'}{r'}\right) |\, 2s > + <1s \,|\, r Y_1^{M_1}\left(\frac{\mathbf{r}}{r}\right) G_{E_{2s}-\omega_2}(\mathbf{r},\mathbf{r}') \tag{1}$$

$$r' Y_1^{M_2 *}\left(\frac{\mathbf{r}'}{r'}\right) |\, 2s > \Big|^2 (\omega_1\omega_2)^3 \, d\omega_2 ,$$

where $G_E(\vec{r}, \vec{r}')$ is the Green's function of the Schrodinger equation for the hydrogen atom. Expanding G_E in spherical functions, we obtain the following solution for the radial Green's function g_ℓ, satisfying the required boundary conditions ($g_\ell \sim r^\ell$ as $r \to 0$ and $g_\ell \to 0$ as $r \to \infty$)

$$g_l(E; r, r') = -\frac{ma\nu}{rr'} \frac{\Gamma(1+l-\nu)}{\Gamma(2l+2)} M_{\nu, l+1/2}\left(\frac{2r_<}{a\nu}\right) W_{\nu, l+1/2}\left(\frac{2r_>}{a\nu}\right), \tag{2}$$

where M and W are Wittaker functions, $r>(<)$ is the larger (smaller) of the quantities r and r', a is the Bohr radius, and $\nu = Z\alpha\sqrt{-m/2E}$ is the "principal quantum number" of the electron in the virtual state; in our problem $1 < \nu < 2$. For the product of the Whittaker functions it is convenient to use an integral representation that is valid when $\nu < \ell + 1$ [4]:

$$M_{\nu, l+1/2}\left(\frac{2r_<}{a\nu}\right) W_{\nu, l+1/2}\left(\frac{2r_>}{a\nu}\right) = \frac{2\sqrt{rr'}}{a\nu} \frac{\Gamma(2l+2)}{\Gamma(l+1-\nu)} \tag{3}$$

$$\int_0^\infty dx \exp\left(-\frac{r+r'}{a\nu} \operatorname{ch} x\right) \left(\operatorname{cth}\frac{x}{2}\right)^{2\nu} I_{2l+1}\left(\frac{2\sqrt{rr'}}{a\nu} \operatorname{sh} x\right),$$

where I is a modified Bessel function of the first kind. Substitution of (2) and (3) in (1) reduces the calculation of the matrix elements to the use of tabulated integrals:

$$dW(x) = \frac{2^8 Z^6 a^8}{3^8 \pi} \frac{mc^2}{\hbar} f(x)\, dx, \tag{4}$$

$$f(x) = \frac{1}{1-x^2}\left[4 \sum_{l=1}^{2} \frac{\nu_l - 1}{\nu_l^2} \, {}_2F_1(1, -1-\nu_l; 3-\nu_l; -\beta_l) - 1\right]^2, \tag{5}$$

where ${}_2F_1$ is a hypergeometric function

$$\beta_l = \frac{(\nu_l - 1)(2-\nu_l)}{(\nu_l + 1)(\nu_l + 2)}$$

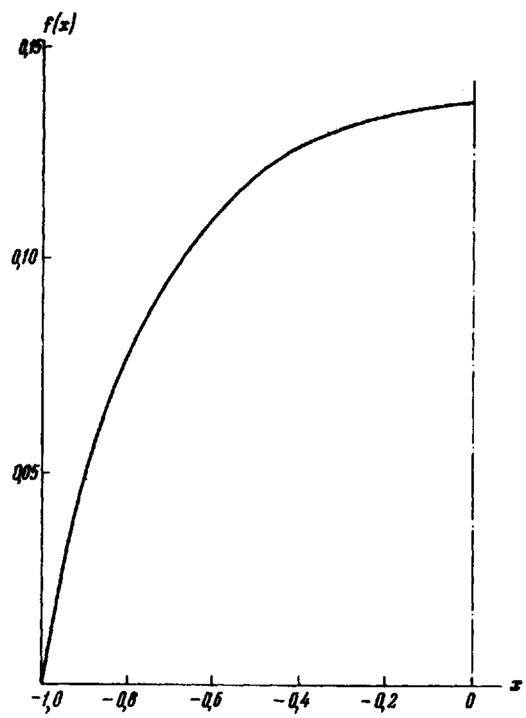
ν_1 and ν_2 are defined in terms of $x(|x| \leq 1)$ as follows: $\nu_1 = \sqrt{8/(5 - 3x)}$ and $\nu_2 = \sqrt{8/(t + 3x)}$ (here

$$\omega_1 = \frac{1}{2}(E_{2s} - E_{1s})(1 - x),\ \omega_2 = \frac{1}{2}(E_{2s} - E_{1s})(1 + x)).$$

It is easy to see that $\beta \lesssim 1/35$ for all β. Confining ourselves to terms $\sim\beta$ in the expansion of ${}_2F_1$, we get

$$f(x) = \frac{1}{1-x^2}\left[\frac{35-6x-9x^2-8\sqrt{2(5-3x)}}{11-9x+\sqrt{2(5-3x)}} + \frac{35+6x-9x^2-8\sqrt{2(5+3x)}}{11+9x+\sqrt{2(5+3x)}} - 1\right]^2. \quad (6)$$

The accuracy of this expression is better than 1%.



Thus, the spectrum is given by formulas (4) - (6) (see the figure). In view of the equivalence of the photons, the total probability of the transition per unit time is equal to

$$W = \frac{1}{2}\,\frac{2^8 Z^6 \alpha^8}{3^8 \pi}\,\frac{mc^2}{\hbar}\int_{-1}^{1} f(x)dx \approx 8{,}226\, Z^6 \ \mathrm{sec}^{-1}$$

which agrees with [2].

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* Here $c = \hbar = 1$ and $\alpha \simeq 1/137$.

POSSIBILITY OF IGNITING A TRAVELING LASER SPARK AT BEAM INTENSITIES MUCH BELOW THE BREAKDOWN THRESHOLD

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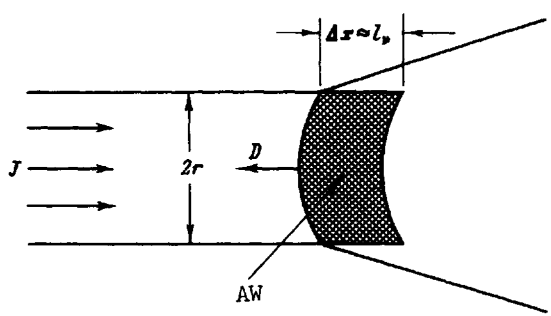
Experimental investigations of the laser spark [1,2] have shown that the boundary of the plasma initially produced in the focus of the lens moves in an optical channel in a direction opposite to that of the laser beam at a velocity ~100 km/sec. The light-absorbing plasma is heated to a temperature higher than half a million degrees [2]. The plasma boundary moving counter to the beam can be regarded as a wave of light absorption and gas heating (AW), which is similar in many respects to the detonation wave in an explosive [3,4].

We wish to emphasize here the circumstance that the tremendous light intensities, approximately 10^5 MW/cm^2, which are produced in the experiments, are not needed at all to maintain the AW, and are required only to produce the initial breakdown in the air. The plasma front absorbing the parallel light beam can propagate without attenuation even at much lower light intensities, far from those needed for breakdown - all that is necessary is to "ignite the detonation," by creating in the light channel an absorbing plasma focus (for example, with the aid of another breakdown-producing laser pulse, a discharge, or some other way).

Let us estimate the lowest limiting intensity J capable of maintaining an undamped AW in a light channel of radius r. The leading front of the AW is a strong shock wave, which ionizes the gas, creating conditions for the absorption of the light. The energy released in the gas, in turn, contributes to the progress of the shock wave. *



The AW width Δx is of the same order of magnitude as the range of the light for absorption in heated gas: $\Delta x = \ell_\nu$. The shock wave is attenuated by energy lost to lateral expansion of the gas in the AW zone (see the figure). The energy loss is very small if $\Delta x \ll r$, but when $\Delta x > r$ it is so large that a self-maintaining AW is impossible in this case. Yet