$$\Delta = \Delta_0 - \frac{\pi\Gamma}{4}; \quad \Gamma = -\frac{2\pi n}{m}\frac{\pi^2 h^2}{p_0}. \tag{2}$$

where Δ_0 is the gap in the absence of impurities. In [2] Γ is positive. In our case $\Gamma < 0$ and therefore the gap increases when the impurities are introduced. Γ is negative because in (1) $D = -1$ when $\omega \sim \Delta$, which in turn, as already noted, is in essence a reflection of the fact that when $g < 0$ the scattering amplitude in the normal metal is maximal when $\omega = 0$. Since we are considering the case $\Delta \ll \epsilon_0$, where ϵ_0 is the characteristic energy starting with which phenomena connected with the Kondo effect [3] come into play, the presence of superconductivity should not affect the existence of this maximum. It turns out in exactly the same manner that in this case the temperature of the transition to the superconducting state also increases. We note also that when $g > 0$ the results obtained by the method indicated above coincide with the corresponding results of [2].

In conclusion, the author is grateful to S. V. Maleev for a large number of interesting discussions.

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TRIANGULAR DIAGRAM IN THE DECAY $Y_0(1520) \to \Lambda\pi\pi$ AND THE DETERMINATION OF THE PION SCATTERING LENGTH

V. V. Anisovich, E. M. Levin, and A. K. Likhoded
A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences
Submitted 10 November 1967
ZhETF Pis'ma 7, No. 3, 99-101 (5 February 1968)

A study of the $Y_0(1520) \to \Lambda\pi\pi$ decay affords a good opportunity of determining the value of a_0 (the scattering length of pions with zero isospin). This decay proceeds in the main in two stages: $Y_0(1520) \to Y_1(1385) + \pi \to \Lambda\pi\pi$ [1]. Therefore the contribution of the triangular diagram (Fig. 1a) to it should be appreciable if the value of a_0 is not small [2-5].

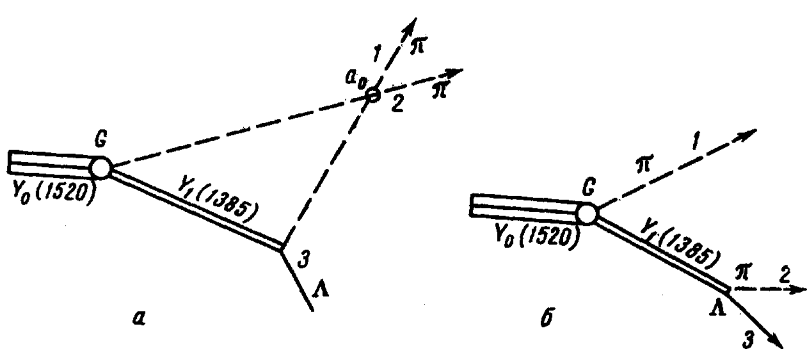

Fig. 1

The logarithmic singularity of this diagram leads to a characteristic variation of the spectrum of the two pions in the $Y_0(1520) \to \Lambda\pi\pi$ decay.

The decay process is determined by the diagrams shown in Figs. 1a and 1b, and the square of the matrix element is:

$$M^2(Y_0(1520)\to\Lambda\pi\pi) = G^2\{\gamma_1|D(13)|^2 + \gamma_{12}\,\mathrm{Re}\,(D^*(13)D(23)) + \gamma_2|D(23)|^2 + \delta_1\,\mathrm{Re}\,(T^*(12), D(13)) + \delta_2\,\mathrm{Re}\,(T^*(12)D(23)) + \delta_0|T(12)|^2\}, \quad (1)$$

where $D(i3) = 1/S_{i3} - I^2$, $I^2 = \Delta^2 + i\Gamma$ is the square of the $Y_1(1385)$ mass, s_{i3} is the square of the total energy of the third and i-th particles in their c.m.s.,

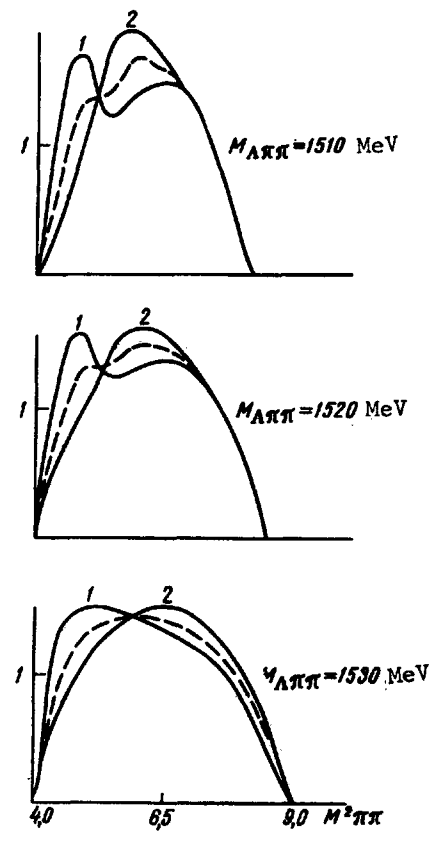

Fig. 2. $\pi\pi$ spectra in the $Y_0(1520) \to \Lambda\pi\pi$ decay at different values of the total system energy $M_{\Lambda\pi\pi} \equiv W$ within the limits of the Y_0(1520 resonance width. Curve 1 corresponds to $a_0 = +1$, curve 2 to $a_0 = -1$, and the dashed curve to $a_0 = 0$.

$$\gamma_i = k_i^2 + 2\kappa_i(\mathbf{k}_1, \mathbf{k}_2) + \kappa_i \mathbf{k}_j^2;$$

$$\delta_i = -[(1+\kappa_i)(\mathbf{k}_1\mathbf{k}_2) + k_i^2 + \kappa_i \mathbf{k}_j^2];$$

$$\gamma_{12} = (1+\kappa_1\kappa_2)(\mathbf{k}_1, \mathbf{k}_2) + \kappa_1 k_2^2 + \kappa_2 k_1^2; \quad \delta_0 = k_3^2;$$

$$\kappa_i = \frac{-W^2 + 1 - S_{i3} + (W/\sqrt{S_{i3}})(S_{i3} + 1 - m^2)}{(W + \sqrt{S_{i3}})^2 - 1};$$

$i, j = 1, 2$; $i \neq j$; $m_\pi = 1$ is the pion mass, and m is the Λ mass.

$$T(12) = \frac{i a_0 h}{2\beta}\left[\ln\frac{a + \sqrt{S_{12}-4}}{|a|} + J\right];$$

$$h = -\frac{m+2}{2(m+1)} + \frac{ma}{4(m+1)}\sqrt{\frac{m+2}{m(W-m-2)}};$$

$$a\beta = \frac{1}{2}W^2 + \frac{1}{2}m^2 - 1 - I^2; \quad \beta = p_3 \text{ for } S_{12} = 4.$$

$$J = -\frac{\Gamma}{2\pi}\int_{m+1}^{W-1}\frac{dS_{13}}{2W}\,\frac{1}{|S_{13} - I^2|^2}\ln\frac{S_{13} - 2p_{30}(W - p_{20}) + 2p_3p_2}{S_{13} - 2p_{30}(W - p_{20}) - 2p_3p_2}.$$

$\vec{k}_i$ and $\vec{p}_i$ are the momenta of the particles in the c.m.s. of the entire reaction and of the two pions, respectively.

The spectrum of the pions at different values of $M_{\Lambda\pi\pi} \equiv W$ is shown in Fig. 2. We see that when $a_0 = +1$ the spectrum has a characteristic maximum at small $M_{\pi\pi}$. The contribution of the triangular diagram can be determined by investigating not only the total pion spectrum, but also the Dalitz diagram. In particular, the characteristic effects can be observed by considering the distribution of the number of events along the resonance (along the line $M_{\pi\Lambda} = 1385$ MeV).

In conclusion, we wish to call attention to the rather single-valued character of (1). This is connected with the fact that the decay into the $Y_1(1385)$ and a pion occurs only in an s-wave and is described by one isotopic amplitude, owing to the quantum numbers of the $Y_0(1520)$ resonance ($T = 0$, $J^P = 3/2^-$) and the low kinetic energy of $Y_1(1385)$.

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ELEMENTARY SOLUTIONS OF THE QUANTUM PROBLEM OF THE MOTION OF A PARTICLE IN THE FIELD OF TWO COULOMB CENTERS

Yu. N. Demkov
Leningrad State University
Submitted 27 November 1967
ZhETF Pis'ma 7, No. 3, 101-104 (5 February 1968)

The problem of the bound states of a particle in the field of two Coulomb centers has in some particular cases simple analytic solutions. These solutions can serve as a check on numerical calculations (similar calculations have been made recently in connection with mesic-molecule problems [1]). In addition, interest attaches to the very existence of such solutions for a problem that plays just as fundamental a role as the problem of the hydrogen atom in atomic physics. This problem has been under study for more than forty years, and it is surprising that these solutions were apparently not discovered earlier.

We denote the charges of the Coulomb centers by Z_1 and Z_2, and the distance between them by R. We shall assume that $Z_1 > 0$ and $Z_2 > Z_1$ and, in addition, that Z_1 and Z_2 are mutually prime integers. The common factor can be readily eliminated by a scale transformation. Then the variables separate in the ellipsoidal coordinates $\xi = (r_1 + r_2)/R$, $\eta = (r_1 - r_2)/R$, and φ. Then, representing the wave function in the form $\psi = F(\xi)G(\eta)\exp(im\varphi)$, we obtain for F an equation containing by way of a parameter only $Z_1 + Z_2$, and for G only $Z_1 - Z_2$. It follows therefore that if the energy of the system (without the interaction of the nuclei) is equal to $(Z_1 + Z_2)^2/2n_1^2$ ($n_1 = 1, 2, \ldots$) and the number of nodes of the function F in the interval $(1, +\infty)$ does not exceed $n_1 - |m| - 1$, then F corresponds to the one-center problem, i.e., it can be represented in the form

$$F = (\xi^2 - 1)^{m/2} P(\xi) \exp[-R(Z_1 + Z_2)\,\xi/2n_1], \qquad (1)$$

where P is a polynomial. The same holds true for the function G, if the energy is equal to $(Z_1 - Z_2)^2/2n_2^2$ and the number of nodes of G in the interval $(-1, +1)$ is not larger than $n_2 - |m| - 1$. If both conditions are simultaneously satisfied, then the two-center problem