INFLUENCE OF THE SECOND UNIAXIAL-ANISOTROPY CONSTANT ON THE MAGNETIZATION CURVE OF A UNIAXIAL ANTIFERROMAGNET WITH WEAK FERROMAGNETISM

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Submitted 7 December 1967; resubmitted 3 January 1968
ZhETF Pis'ma 7, No. 4, 142-147 (20 February 1968)

The thermodynamic theory of weak ferromagnetism was first constructed by Dzyaloshinskii with hematite (α-Fe_2O_3) as an example [1]. The properties of hematite in the antiferromagnetic state (below the transition point T_K = 253°K) and for a transition in a field from the antiferromagnetic to the weakly-magnetic state were investigated both theoretically and experimentally in the subsequent papers [2-7]. The authors of the theoretical papers [6,7] considered a uniaxial antiferromagnet with a Dzyaloshinskii interaction and took into account only the first uniaxial-anisotropy constant. However, investigations of hematite in strong magnetic fields [4,5] have shown that the theoretical equations of [6,7] do not agree with the experimentally obtained dependences of the magnetization of hematite in the basal plane ($m_\perp$) on the field in the basal plane ($H_\perp$) (for $T < T_K$). In [6,7] they predicted a linear dependence of $m_\perp$ on $H_\perp$, whereas the experimental curves are nonlinear. Owing to the nonlinearity of the magnetization curves, the experimental values of the fields of transition from the antiferromagnetic to the weak-ferromagnetic state upon magnetization in the basal plane are smaller than those calculated theoretically [4].

According to a hypothesis advanced in [6], the magnetization curve $m_\perp(H_\perp)$ near the transition point is strongly influenced by the second constant of uniaxial anisotropy.

We shall show below that Dzyaloshinskii's thermodynamic theory [1] explains the experimental magnetization curves of hematite, provided the second uniaxial-anisotropy constant is taken into account, and that allowance for this constant is essential at temperatures much lower than T_K.

The expansion of the thermodynamic potential in this case is

$$\Phi = \frac{a}{2}\sin^2\theta + \frac{b}{4}\sin^4\theta + \frac{B}{2}m^2 - qm_\perp\sin\theta + \frac{c}{2}m_\parallel^2 + \frac{D}{2}m_\parallel^2\cos^2\theta - \\ - m_\perp H_\perp - m_\parallel H_\parallel. \qquad (1)$$

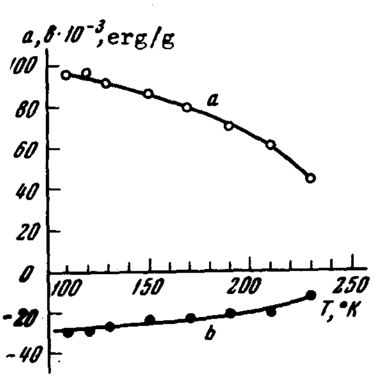

Fig. 1. First uniaxial-anisotropy constant a (open circles) and second uniaxial-anisotropy constant b (full circles) vs. the temperature.

Here $(a/2)\sin^2\theta + (b/4)\sin^2\theta$ is the energy of the uniaxial anisotropy, where a and b are the first and second uniaxial-anisotropy constants and θ is the angle between the direction of the antiferromagnetism vector and the trigonal axis; $(B/2)m^2$ is the second-order exchange term; $qm_\perp \sin\theta$ is the "mixed" term responsible for the weak ferromagnetism; $m_\perp$ and $m_\parallel$ are the components of the magnetic moment m in the basal plane and along the trigonal axis, respectively; $(c/2)m_\parallel^2$ is the second-order magnetic-anisotropy term, and $(D/2)m_\parallel^2\cos^2\theta$ is the fourth-order exchange term; $H_\perp$ and $H_\parallel$ are the components of the magnetic field intensity in the basal plane and along the trigonal axis, respectively.

In the expansion of the thermodynamic potential (1), we neglected the anisotropy in the basal plane, since it is insignificant (the effective anisotropy field in the basal plane does not exceed 1 Oe [8]).

For magnetization along the trigonal axis we obtain from (1)

$$m_\parallel = \frac{H_\parallel}{B + c + D\cos^2\theta}, \tag{2}$$

where

$$\cos\theta = 0 \quad \text{for} \quad H_\parallel \geq H_K \tag{3}$$

and

$$\cos\theta = 1 \quad \text{for} \quad H_\parallel \leq H_K. \tag{4}$$

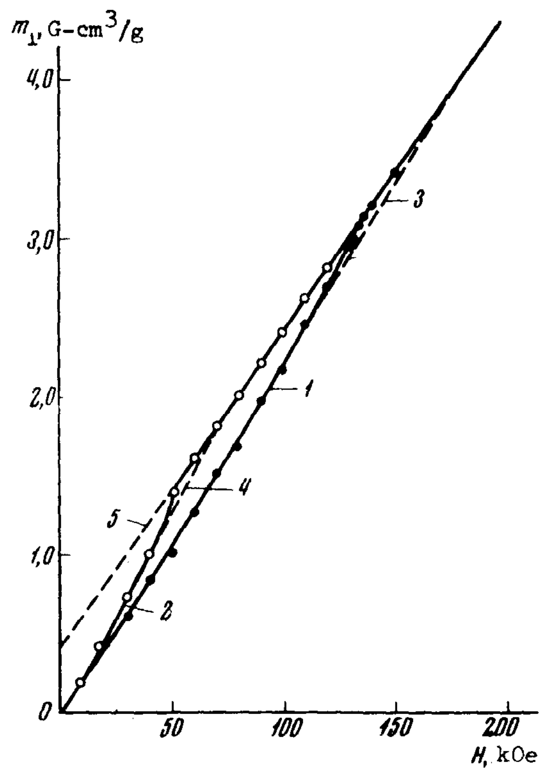

Fig. 2. Magnetization in the basal plane vs. the field H.
• - 120°K, o - 230°K: experimental points;
1 - 120°K, 2 - 230°K: theoretical relations with allowance for both anisotropy constants;
3 - 120°K, 4 - 230°K: theoretical relations with allowance for one uniaxial-anisotropy constant;
5 - magnetization in the weakly-ferromagnetic state (above the transition point).

Here H_K is the field of the first-order phase transition from the antiferromagnetic state to the weakly-magnetic state in magnetization along the trigonal axis:

$$H_K = [(a - \frac{q^2}{B} + \frac{b}{2})(B + c)(1 + \frac{B + c}{D})]^{1/2}. \tag{5}$$

For magnetization in the basal plane we get from (1)

$$m_\perp = \frac{q \sin\theta + H_\perp}{B}, \tag{6}$$

where

$$\sin\theta = 1 \quad \text{when } H_\perp \geq H_0 \tag{7}$$

and

$$(a - \frac{q^2}{B}) \sin\theta + b \sin^3\theta = \frac{q}{B} H_\perp \qquad \text{when } H_\perp \leq H_0 \tag{8}$$

From (6) and (8) we see that allowance for the second-anisotropy constant leads to a nonlinear dependence of the magnetization in the basal plane on the field. Analysis of (6) and (8) shows also that the transition from the antiferromagnetic into the weakly-magnetic state in the field $H_\perp$ can occur either smoothly (second-order phase transition) or jumpwise (first-order phase transition), depending on the ratio of the anisotropy constants a and b, and also of the parameters q and B.

If $a > 0$ and $b < 0$, as is the case for hematite at $T < T_K$, then under the condition

$$\frac{a - q^2/B}{3|b|} \geq 1 \tag{9}$$

the transition from the antiferromagnetic to the weakly-ferromagnetic state in the field $H_\perp$ is smooth (second-order phase transition) and the transition field is equal to

$$H_0^{II} = \frac{B}{q}(a - q^2/B + b), \tag{10}$$

If

$$\frac{a - q^2/B}{3|b|} \leq 1 \tag{11}$$

the transition from the antiferromagnetic into the weakly-magnetic state in the field $H_\perp$ occurs jumpwise (first-order phase transition) and the transition field is equal to

$$H_0^I = \frac{2}{3} \frac{B}{q} \sqrt{-\frac{(a - q^2/B)^3}{3b}}. \tag{12}$$

We note that hysteresis occurs in the case of a first-order phase transition and the reverse transition from the weakly-ferromagnetic into the antiferromagnetic state should occur at a different field value, H_0^{II}. Since the experiment yields directly the values of the fields of the transition H_K and H_0, it is more convenient to express conditions (9) and (11) in the form

Second order transition $$1{,}25 \frac{H_0}{H_K^2} \cdot \frac{q}{B} (B + c)\left(1 + \frac{B + c}{D}\right) \geq 1. \qquad (13)$$

First order transition $$1{,}25 \frac{H_0 q}{H_K^2 B} (B + C)\left(1 + \frac{B + c}{D}\right) \leq 1. \qquad (14)$$

According to our measurements of the magnetization [4], the values of the parameters are: $B = 5.2 \times 10^4$ g/cm^3, $q = 2.2 \times 10^4$ G, $c = 6 \times 10^3$ g/cm^3, and $D = 16.5 \times 10^5$ g/cm^3. Using the values of the transition fields H_K and H_0 from [4] we can assess, from the conditions (13) and (14), the character of the transition from the antiferromagnetic to the weakly-ferromagnetic state in the field $H_\perp$. Our calculations show that above 200°K this transition is of first order. Below this temperature, the quantity

$$1{,}25 \frac{H_0 q}{H_K^2 B} (B + c)\left(1 + \frac{B + c}{D}\right) \qquad (15)$$

is equal to 1 within the limits of error, so that the character of the phase transition at $T \lesssim 200$°K cannot be determined.

Figure 1 shows plots of the anisotropy constants a and b against the temperature, as calculated from formulas (5) and (10) (or (12)), using the values of the parameters B, q, c, and D given above and the values of H_0 and H_K from [4]. In the temperature interval 110 - 230°K, the ratio of the second anisotropy constant to the first is ≈ -0.3.

Figure 2 shows the theoretical plots of the magnetization in the basal plane with allowance for two anisotropy constants, and the experimental values of the magnetization from [4]. For comparison it shows also the theoretical $m_\perp(H_\perp)$ plot with account of only one anisotropy constant. It is seen that allowance for the second anisotropy constant leads to good agreement between theory and experiment, and that this allowance is essential at temperatures much lower than the transition temperature.

We are grateful to Professor K. P. Belov for interest in the work, and to A. S. Borovik-Romanov, A. M. Kadomtsev, V. I. Ozhogin, and A. S. Pakhomov for valuable discussions.

[1] I. E. Dzyaloshinskii, Zh. Eksp. Teor. Fiz. 32, 1547 (1957) [Sov. Phys.-JETP 5, 1259 (1957)].
[2] P. Flanders and S. Shtrikman, Solid State Comm. 3, 285 (1965).
[3] R. A. Voskanyan, R. Z. Levitin, and V. A. Shchurov, Zh. Eksp. Teor. Fiz. 53, 459 (1967) [Sov. Phys.-JETP 26, 302 (1968)].
[4] R. A. Voskanyan, R. Z. Levitin, and V. A. Shchurov, ibid. 54, No. 3 (1968) [27 (1968), in press].
[5] G. Cinader and S. Shtrikman, Solid State Comm. 4, 459 (1966).
[6] V. I. Ozhogin and V. G. Shapiro, ZhETF Pis. Red. 6, 467 (1967) [JETP Lett. 6, 7 (1967)].
[7] K. Mizushima and S. Iida, J. Phys. Soc. Japan 21, 1521 (1966).