

these properties of the collision term, we can obtain from (2) in the standard manner the hydrodynamic equations describing the longitudinal motion of the plasmoid. The result coincides exactly with the hydrodynamic equations for an ideal gas with an adiabatic exponent $\gamma = (3 + 2\alpha)/(1 + 2\alpha)$.

In our problem we are interested in those solutions of those hydrodynamic equations which correspond to the spreading of a gas cloud in vacuum. An investigation of such a problem can be found, for example, in the book of Zel'dovich and Raizer [2]. It turns out that when $t \gg L^{(0)}/v_{Ti}^{(0)}$ the boundaries of the plasmoid move apart at a constant velocity $\sim v_{Ti}^{(0)}$, and the transverse temperature of the ions decreases like

$$T_{\perp i} \sim T_{\perp i}^{(0)} \left(\frac{L^{(0)}}{v_{Ti}^{(0)} t} \right)^{\gamma-1}.$$

The foregoing investigation pertains to the case when $T_{\parallel i} \gg T_e$. However, a similar effect can exist also when $T_{\parallel i} \ll T_e$, for in this case the spreading of the plasma cloud leads to a decrease in the longitudinal thermal scatter of the ions [3]. When $T_{\parallel i} \ll T_e$ the isotropization of the ions is due to the excitation of oscillations of the ion-sound type. We note that the corresponding increments can greatly exceed the value $\sqrt{m/M} \omega_{Hi}$.

The described effect can be used in experiments on charge exchange of a plasmoid with a gas target. The atoms produced during the charge exchange leave the target at an angle $\sim v_{\perp}/v_{\parallel}$ to the plasmoid direction of motion. Therefore the decrease of the transverse ion velocity as a result of the "conversion" of the transverse energy into longitudinal-motion energy will cause the divergence of the atom plasmoid to decrease greatly compared with the case when the charge exchange is effected near the injector and the cyclotron instability does not have time to develop.

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NONSTATIONARY NONLINEAR OPTICAL EFFECTS AND FORMATION OF ULTRASHORT LIGHT PULSES

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1. An important recent accomplishment in laser physics is the generation of picosecond giant pulses (of duration $\tau_p \approx 10^{-11}$ sec for single pulses and $\tau_p \approx 10^{-12}$ for pulse trains, see [1]). Inasmuch as the reduction of the pulse duration leads to an increase of the threshold of the light-field intensity E_{br} at which breakdown of the medium takes place (see

[2,3], $E_{br} \sim 1/\sqrt{\tau_p}$ for an electron cascade with single pulses), the "dynamic range" of the nonlinear optics is broadened. It would be particularly interesting to obtain single pulses with a limiting duration determined by the dispersion spreading and by nonstationary diffraction ($\tau_p \sim 10^{-14}$ sec in the visible band and $\tau_p \sim 10^{-15}$ sec in the ultraviolet); then one could speak of production of strong nonlinear effects (so far we have referred to "weak" nonlinearity, see [4]) with an electronic nonlinearity (of the type of optical shock waves), or of "instantaneous" inversion in systems with short lifetimes (which may lead to the development of lasers for the far ultraviolet, etc.).

We consider in the present letter the singularities of nonlinear wave interactions in the field of an ultrashort pulse. It is shown below that the wave interactions involving electronic nonlinearities are a very effective means of producing giant light pulses of duration reduced to the limit. Three-photon parametric interactions are in prospect; the reduction of the pulse duration can be accompanied here by a noticeable growth of the peak power.

2. Nonlinear optical effects at $\tau_p \approx 10^{-11}$ have, generally speaking, a nonstationary character. The nonstationary nature can be connected with the non-quasistatic nature of the local nonlinear response (the nonlinearity relaxation time is $\tau \geq \tau_p$) and the nonquasistationary nature of the response of the system under consideration as a whole (the time of the group delay of the interacting wave in the medium is $\tau_d = l(1/u_1 - 1/u_2)$, $\tau_d > \tau_p$, see [5,6]). For nonlinear pulse shaping we should obviously have $\tau_p > \tau$, so that further narrowing of picosecond pulses cannot be obtained with laser amplifiers (see [7,8]), with the high-frequency Kerr effect (see [9-11]), with stimulated Raman scattering (where $\tau \approx 10^{-12}$ sec, see [12]), or even with stimulated Mandel'shtam-Brillouin scattering [11]. For the electronic nonlinearity, on the other hand, $\tau \sim 10^{-15}$ sec.

3. Let us turn to consider three-photon interactions of short pulses. Frequency-degenerate three-photon interaction of plane wave packets (frequency doubling and degenerate parametric amplification) whose average frequencies satisfy the phase-synchronism condition is described by the system

$$\frac{\partial A_1}{\partial Z} + \frac{i}{2} \frac{\partial^2 k_1}{\partial \omega^2} \frac{\partial^2 A_1}{\partial \eta^2} + \delta_1 A_1 = \sigma A_2 A_1^*, \quad \eta = t - z/u_1; \quad (1)$$

$$\frac{\partial A_2}{\partial Z} + \beta \frac{\partial A_2}{\partial \eta} + \frac{i}{2} \frac{\partial^2 k_2}{\partial \omega^2} \frac{\partial^2 A_2}{\partial \eta^2} + \delta_2 A_2 = \sigma A_1^2, \quad \beta = \frac{1}{u_2} - \frac{1}{u_1}. \quad (2)$$

Here A_1 and A_2 are the complex amplitudes of the waves at frequencies ω and 2ω ; $u_1 = \partial\omega/\partial k$ are the group velocities, and $\sigma = 2\pi\chi\omega^2/c^2 k_1$, where χ is the tensor of the quadratic nonlinear polarization. An analysis of the system (1) and (2) (for which we obtained, in the case when $z < \tau/(\partial^2 k/\partial \omega^2)$ and $\delta_2 = 0$ (see also [13,14]), exact solutions that take into account both the group delay and the strong energy conversion from one wave into another) leads to the following results.

4. In the linear mode of parametric amplification ($A_2(z) \approx A_2(0) \equiv E_2$) it is possible to obtain narrower pulses if the conditions for phase and group synchronism are simultaneously

satisfied.*

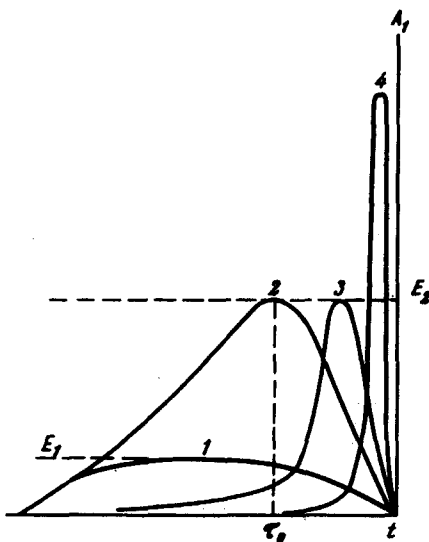
In a pulsed pumping field with $\sigma E_2 z \gg 1$ the amplified signal is also a pulse. A system consisting of a frequency doubler followed by a parametric converter (triggered by a pulse of the fundamental frequency or by parametric luminescence, see [15]), retains the same average frequency and reduces the width of a Gaussian pulse in a ratio

$$\kappa = \frac{r_{\text{out}}}{r_{\text{in}}} = \sqrt{\frac{\beta}{2\sigma E_{\text{br}}(\delta_{\text{in}}) r_{\text{in}}}}.$$

For an Nd^{3+} laser with $\tau_{\text{in}} \approx 10^{-11}$ sec and breakdown power density $P_{\text{br}} \approx 4 \times 10^9$ W/cm², with KDP and BaNbO_3 crystals, it is possible to obtain $\kappa \approx 0.15$ in one stage, with a power efficiency on the order of $10^{-1} - 10^{-2}$.

5. Appreciable narrowing of parametrically amplified pulses in the field of quasi-continuous pumping can be obtained in an essentially nonlinear amplification mode. In the presence of group detuning, the character of the energy exchange between the subharmonic and the pump is different in this case for the leading and trailing edges of the pulse. If $u_1 > u_2$, the leading front can enter into the exponential-amplification mode even when $A_1 \sim A_2$; then practically all the pump power is absorbed by the leading edge, and the trailing edge becomes much sharper as a result of the nonlinear loss due to the frequency doubling. We present the main results of the theory of the effect. The solution of the system (1) and (2), neglecting the dispersion spreading, with boundary conditions $A_1(0, t) = E_1 \Phi(t)$, $A_2(t, z) \approx E_2$ for $E_2 > E_1$ takes the following form:

$$A_1(t, z) = \frac{E_1 \Phi(t - z/u_1) \exp(\sigma E_2 z)}{1 + \left(\sigma \frac{E_1^2}{E_2}\right) \int_0^z \Phi^2\left[t - \frac{z}{u_2} + \beta y\right] \exp(\sigma E_2 y) \text{sh } \sigma E_2 y dy} \quad (3)$$



Dynamics of the shaping of an ultrashort subharmonic pulse in a quasi-continuous pumping field when $\tau_0 > \tau_{\text{cr}}$: 1 - pulse at the entrance to the nonlinear medium, 2 - pulse at a distance $z = z_f$ from the entrance ($\tau_1 \sim \tau_{\text{cr}}$, $A_1 \approx E_2$), 3 - $z \approx 2z_f$, 4 - $z > 2z_f$, subharmonic amplitude increases exponentially.

An analysis of formula (3) (case $\beta > 0$) shows that:

a) A pulse with an exponential leading edge $\Phi = \exp(t/\tau_0)$ produces at a length

$$z_f = z_{nl} \ln \left[\frac{2E_2}{E_1} \sqrt{1 + r_{cr}/r_0} \right]$$

a stationary pulse in the form

$$E_{st} \sim E_2 \sqrt{1 + r_{cr}/r_0} / \text{ch}(t/r_0),$$

with a peak having a velocity $v > u_1$. In the foregoing expressions $\tau_{cr} = \beta z_{nl}$, where $z_{nl} = (\sigma E_2)^{-1}$. Thus, for the pulse under consideration, the slope of the leading edge remains unchanged. The narrowing is due to the deformation of the trailing edge.

b) In order to increase appreciably the slopes of the edges, the leading edge of the initial pulse must be steeper than exponential. In particular, it is possible to shape from a pulse with a linear leading edge $\Phi = t/\tau_0$ a pulse whose duration is limited in principle only by the relaxation time of the electron polarization and by the dispersion spreading. The development of the shaping process depends in this case on the ratio τ_{cr}/τ_0 . When $\tau_0 > \tau_{cr}$ a leading front of duration $\sim \tau_{cr}$ is shaped at first, at a distance

$$z_f = z_{nl} \ln \left[\frac{2E_2}{E_1} \sqrt{\frac{r_0}{r_{cr}}} \right]$$

The amplitude of the subharmonic reaches the pump amplitude; the trailing edge is shortened (see the figure). After a distance $\sim 2z_f$ is covered, exponential growth of the subharmonic amplitude begins on the leading edge (the subharmonic experiences "fresh" pumping), in accordance with the law

$$A_1 = E_2 \exp \frac{z - 2z_f}{z_{nl}}.$$

The narrowing of the leading edge is now in accordance with the law

$$r_1 = \exp \left(- \frac{z}{z_{nl}} \right) \frac{E_2}{E_1} \sqrt{r_0 r_{cr}},$$

and that of the trailing edge

$$r_2 = \exp \left(- \frac{2z}{z_{nl}} \right) r_{cr} \left(\frac{E_2}{E_1} \right)^2.$$

It must be emphasized that although the described picture is similar to the mechanism of line narrowing in a laser amplifier (see [7,8]) or in an SRS amplifier (see [12]), the use of parametric systems has important advantages. It must also be mentioned that besides the short relaxation time of the nonlinearity and the presence of a thresholdless nonlinear-loss

mechanism, parametric amplifiers make it possible to use picosecond Gaussian triggering pulses from synchronized-mode lasers, or pulses shaped in self-focusing liquids (if $\beta < 0$). When $E_1 \geq 10^4$ cgs esu, the weak "tails" of the signal pulse can be suppressed by the nonlinear addition to the phase velocity.

It is also possible to use ganged systems including saturating filters. In birefringent crystals (such as KDP) it is possible to obtain both $\beta > 0$ and $\beta < 0$. Estimates show that in a KDP crystal of length $l \approx 2 - 3$ cm and in a pump field with $\tau_2 \approx 10^{-9} - 10^{-10}$ sec and $\lambda_2 \approx 0.53 \mu$, the pulse with $\lambda_1 \approx 1.06 \mu$ and $\tau_p \approx 10^{-11}$ sec can narrow down to $\tau_{cr} \approx 10^{-13}$ sec at $P_2 \approx 10^9$ W/cm² (here $\beta \approx -3 \times 10^{-13}$ sec/cm). Then a pulse of limiting duration is shaped over a distance of the same order. Here $\beta \approx 3 \times 10^{-12}$ in the $0.53 - 0.26 \mu$ region, $\tau_p < \tau_{cr}$ even for ordinary picosecond pulses, and the total shaping length is reduced.

We note in conclusion that the narrowing is possible also in the nondegenerate amplification mode; interest attaches here to cases when one of the frequencies is strongly absorbed. Multiplication of the frequency of picosecond pulses is discussed in [14].

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* In the direction specified here, the condition of group synchronism can be assumed to be approximately satisfied at lengths $z < \tau_p/\beta$; we note that in some of the crystals there exist directions of exact group synchronism, which can be aligned, by proper choice of the external conditions, with the directions of phase synchronism.