DISSIPATIVE-PARAMETRIC BUILDUP OF OSCILLATIONS WITH ANOMALOUS DISPERSION IN A PLASMA SITUATED IN A HIGH-FREQUENCY FIELD

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A plasma located in a strong high-frequency field is unstable, under certain conditions, to buildup of a potential electric field. The theory of parametric resonance in such a plasma predicts the existence of a hydrodynamic instability [1] in relatively strong fields at frequencies close to the Langmuir frequency of the electron and less, and also the existence of a kinetic instability [2] due to the high-frequency Cerenkov effect on the electrons, in a non-isothermal plasma at external-field frequencies higher than the electron Langmuir frequency. In this communication we wish to point to the existence of one more instability, which differs qualitatively in character from those considered in [1,2]. Such an instability can occur in relatively weak fields, when the velocity of the oscillations of the electron is small compared with its thermal velocity. It can therefore be investigated experimentally with relative ease, and furthermore can lead to new qualitatively important effects, wherein a relatively weak high-frequency electromagnetic field experiences anomalously large turbulent absorption.

As follows from the results of [2], when the frequency of the external field decreases and approaches the electron Langmuir frequency, the wavelength of the growing oscillations increases and the threshold electric-field intensity at which the kinetic instability is possible decreases. Therefore, being interested in the region of electric field frequency ω_0 close to the limit of the plasma transparency region, we can assume that the oscillation wavelengths are larger than the electron Debye radius (r_{De}) or the electron oscillation amplitude in the external field (r_E). Under these conditions, the dispersion equation of the longitudinal plasma oscillations in a homogeneous hf electric field (see [1])

$$0 = D(\omega + i\gamma, \mathbf{k}) = \frac{\epsilon(\omega + i\gamma, \mathbf{k})}{[1 + \delta\epsilon_e(\omega + i\gamma, \mathbf{k})]\delta\epsilon_i(\omega + i\gamma, \mathbf{k})} + \frac{(\mathbf{k}r_E)^2}{4}\frac{\epsilon(\omega_0 + \omega + i\gamma, \mathbf{k}) + \epsilon(-\omega_0 + \omega + i\gamma, \mathbf{k})}{\epsilon(\omega_0 + \omega + i\gamma, \mathbf{k})\,\epsilon(-\omega_0 + \omega + i\gamma, \mathbf{k})}, \quad (1)$$

for oscillations with a small increment ($\omega \gg \gamma$), yields

$$\left\{1 - \frac{1}{2}\gamma^2\frac{\partial^2}{\partial\omega^2}\right\}\left[\frac{1}{\delta\epsilon_i'(\omega, \mathbf{k})} + \frac{1}{1 + \delta\epsilon_e'(\omega, \mathbf{k})}\right] + \frac{1}{4}(\mathbf{k}r_E)^2\left[\frac{u_+}{v_+} + \frac{u_-}{v_-}\right] = 0, \quad (2)$$

$$\gamma = \frac{-1}{\Xi}\left\{\frac{\delta\epsilon_e''(\omega, k)}{[1+\delta\epsilon_e'(\omega,k)]^2} + \frac{\delta\epsilon_i''(\omega, k)}{[\delta\epsilon_i'(\omega,k)]^2} + \frac{1}{4}(kr_E)^2\left[\frac{\delta\epsilon_e''(\omega_0+\omega,k)}{v_+} + \frac{\delta\epsilon_e''(-\omega_0+\omega, k)}{v_-}\right]\right\}. \tag{3}$$

Here $\delta\epsilon_\alpha'(\omega, \vec{k})$ and $\delta\epsilon_\alpha''(\omega, \vec{k})$ are the real and imaginary parts of the partial contribution of particles of type α to the longitudinal dielectric constant $\epsilon(\omega, \vec{k}) = 1 + \delta\epsilon_e(\omega, \vec{k}) + \delta\epsilon_i(\omega, \vec{k})$, and

$$u_\pm = \epsilon'(\pm\omega_0, k) \pm \omega\partial\epsilon'(\pm\omega_0, k)/\partial\omega_0, \tag{4}$$

$$v_\pm = [u_\pm]^2 + [\epsilon''(\pm\omega_0 + \omega, k) \pm \gamma\partial\epsilon'(\pm\omega_0, k)/\partial\omega_0]^2, \tag{5}$$

$$\Xi = -\frac{\partial}{\partial\omega}\left[\frac{1}{\delta\epsilon_i'(\omega, k)} + \frac{1}{1+\delta\epsilon_e'(\omega, k)}\right] + \frac{1}{4}(kr_E)^2\left[\frac{1}{v_+} - \frac{1}{v_-}\right]\frac{\partial\epsilon'(\omega_0, k)}{\partial\omega_0}. \tag{6}$$

We note that near the transparency threshold we have $|\epsilon'(\pm\omega_0, \vec{k})| \ll 1$. Therefore the terms $\sim r_E^2$ of (2), (3), and (6) become quite important even in a weak field. Similar terms in the numerator of (3) determine the kinetic instability due to the high-frequency Cerenkov effect on the electrons [2]. Under the conditions considered in [2], the function Ξ is positive. We note that if γ can be neglected in (5), then $\Xi = -\partial D'(\omega, \vec{k})/\partial\omega$, where D' is the corresponding real part of (1). Therefore the case $\Xi > 0$ can be called the case of normal dispersion. The qualitatively new possibility arises under conditions of anomalous dispersion, when the function Ξ becomes negative. In this case the oscillations grow even under conditions when the Cerenkov effect on the electrons at frequencies $\omega \pm \omega_0$ is negligible. The latter denotes that we can leave out $\epsilon''(\pm\omega_0 + \omega, \vec{k})$ from (3) and (5).

Let us consider the simplest case of a plasma in the absence of collisions, without an external magnetic field. Then the instability discussed by us will take place in a wide range of parameters for oscillations of frequency

$$\omega = \sqrt{\frac{1}{2}\{[\Delta\omega_0]^2 + \omega_s^2 + ([\Delta\omega_0]^2 - \omega_s^2)\sqrt{1-\phi}\} + \gamma^2}. \tag{7}$$

Here ω_s is the frequency of the ion-acoustic oscillations, $\Delta\omega_0$ is the difference between ω_0 and the frequency of the longitudinal electron oscillations

$$\phi = \frac{1}{4}\frac{(kr_E)^2\omega_{Li}^2\omega_0\Delta\omega_0}{\{[\Delta\omega_0]^2 - \omega_s^2\}^2}$$

and ω_{Li} is the ion Langmuir frequency. Oscillations of frequency (7) take place when their phase velocity is small compared with the electron thermal velocity and is large compared with that of the ions. According to (7), $\varphi < \varphi_{cr} = 1$, for hydrodynamic instability sets in otherwise [1]. For the growth increment of the oscillations (7) we obtain, in accordance with Eq. (3)

$$\gamma = -\frac{2\gamma_0}{\Xi} = \frac{-1}{\Xi}\sqrt{\frac{\pi}{2}}\left\{\frac{\omega_{Li}^2}{\omega_{Le}^2} k v_{Te} + \frac{\omega^4}{k^3 v_{Ti}^3}\exp\left(-\frac{\omega^2}{2k^2 v_{Ti}^2}\right)\right\}, \tag{8}$$

$$\Xi = -\frac{4}{\phi}[\sqrt{1-\phi} + 1]\left[\sqrt{1-\phi} + 0(\gamma^2)\right] \tag{9}$$

It is obvious that under conditions when γ^2 is small, we have $\Xi < 0$ for $\varphi > 0$. The instability therefore occurs in the region of transparency to the field of frequency ω_0. On the other hand, the term $-\gamma^2$ in (9) becomes significant when φ approaches the critical value $\varphi_{cr} = 1$, when Ξ decreases, and the increment increases appreciably. In the vicinity of $\varphi = 1$, where $\partial D'/\partial\omega$ reverses sign, the increment has a maximum, but still remains lower than the frequency ω:

$$\gamma_{max} \approx \frac{\gamma_0}{\sqrt{1-\phi}} \approx |\gamma_0\{[\Delta\omega_0]^2 - \omega_s^2\}|^{1/3} \tag{10}$$

for in this case

$$\sqrt{1-\phi}\,|\gamma_0\{[\Delta\omega_0]^2 - \omega_s^2\}|^{1/3} \approx 1, \tag{11}$$

We note that in this region $\Delta\omega_0$ and ω are close to the ion-sound frequency ω_s, and the second possible solution of (2) is close to (7).

To be able to neglect the high-frequency Cerenkov effect on the electrons it is necessary to satisfy the condition

$$\gamma >> \delta\epsilon_e''(\omega_0, k)/[\partial\delta\epsilon_e'(\omega_0, k)/\partial\omega_0],$$

which yields for φ not close to unity

$$\frac{E^2}{E_T^2} \equiv E^2\frac{e^2}{m\omega_0^2\kappa T} >> \frac{\omega_0}{\Delta\omega_0}\left\{1-\left[\frac{\Delta\omega_0}{\omega_s}\right]^2\right\}^2 \times \times\frac{1}{k^2 r_{De}^2}\exp\left\{-\frac{1}{2k^2 r_{De}^2}\right\}. \tag{12}$$

Simultaneously with this condition, which is the lower bound for the electric field, we can write the corresponding condition characterizing the admissible region of wave numbers; for a hydrogen plasma this yields $kr_{De} < 0.2$. For values of φ close to unity, the field intensities, together with the admissible region of k, is determined by the inequality

$$(\gamma_0/\omega_0) >> \sqrt{1-\phi}(kr_{De})^{-3}\exp\left[-\frac{1}{2}k^{-2}r_{De}^{-2}\right]$$

and by relation (11). In the case $E \ll E_T$ this is possible, for example when $\Delta\omega_0 \approx \omega_s$. We note that in order to satisfy these conditions the plasma must be strongly non-isothermal ($T_e \gg T_i$).

We note in conclusion that particularly favorable conditions for the appearance of the instability in question occur in the presence of a constant magnetic field, similar to the case of the ordinary plasma instability under conditions of anomalous dispersion [3]. It must be assumed that a similar phenomenon can take place also in condensed media.

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[3] A. V. Timofeev and V. I. Pistunovich, in: Voprosy teorii plazmy (Problems of Plasma Theory), ed. by M. A. Leontovich, No. 5, Atomizdat, 1967, p. 351.

AMPLIFICATION OF SHORT-WAVE RADIATION IN A PLASMA OF MULTIPLY CHARGED IONS

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For media that amplify effectively in the vacuum ultraviolet or the soft x-ray band, the requirements of absolute level population inversion are much more stringent than in the visible spectrum. This is due to the weaker reflection from the "mirrors" and to the drop in the probability ratio of the stimulated and spontaneous transitions. When speaking of the use of a plasma as such a medium, mention must be made of the remarks made in [1,2]. In [1] we considered pulsed recombination of nuclei into multiply charged hydrogenlike ions; [2] dealt with conditions created by charge exchange of ions passing through a specially chosen material. The main difficulty in the procedure discussed in [1] is the need for rapidly shutting off the heating field, since the relaxation times of a dense plasma of multiply charged or hydrogen-like ions are quite short. In the scheme proposed by Smirnov in [2], the smallness of the ion relaxation time also leads to a technical problem, where competition sets in between the relaxation times and the times of passage of the ion through the substance. In addition, it is impossible to attain a sufficiently high density in a pure ion beam; on the other hand, in a multiply charged ion plasma beam the high temperature of the free electrons leads rapidly to ionization of the "charge exchanging" substance and to deceleration of the beam.

Unlike the cooling scheme [1] and the beam scheme [2], we shall discuss here the conditions arising when a plasma of two chemical elements is rapidly heated. During the time Δt when the heating field is applied, the atoms B_1 and B_2 are essentially ionized to different ion multiplicities n_1 and n_2. Let us denote by E_1 the ionization energy $B_1^{(n_1-1)+}$ of the ion of multiplicity $n_1 - 1$ and by E_2 the ionization energy $B_2^{(n_2-1)+}$ of the ion of multiplicity $n_2 - 1$; let $E_1 > E_2$. The values of E_1 and E_2 depend strongly, besides on Δt and on the heating-field intensity, also on the structure of the external electron shells of B_1 and