($g_k$ is the statistical weight of the state), and is sufficient in principle to develop a generator in the soft x-ray band; the effect of the decrease of the wavelength $\lambda_{mp}$ compared with lasers is fully compensated by the high probability $A_{mp}$ of the spontaneous emission and by the values $N_p$ of the level populations that are possible in the discussed case.**


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* The energy defect is determined not only by its value at large distances, but also by the Coulomb and polarization interaction of the ions.

** The values of the width of the spectral line $\Gamma_{mp}$ are practically independent of the free-electron concentration, up to very high plasma densities, since the cross sections of the "collision" transitions of the external electron of the multiply charged ion $B^{n+}$ decrease rapidly with increasing n; these cross sections are even smaller for the internal electrons of the ion.

# INFLUENCE OF THE RELATIVE DISPLACEMENT OF THE MINIMA OF THE VALLEYS IN A MAGNETIC FIELD ON OSCILLATORY EFFECTS


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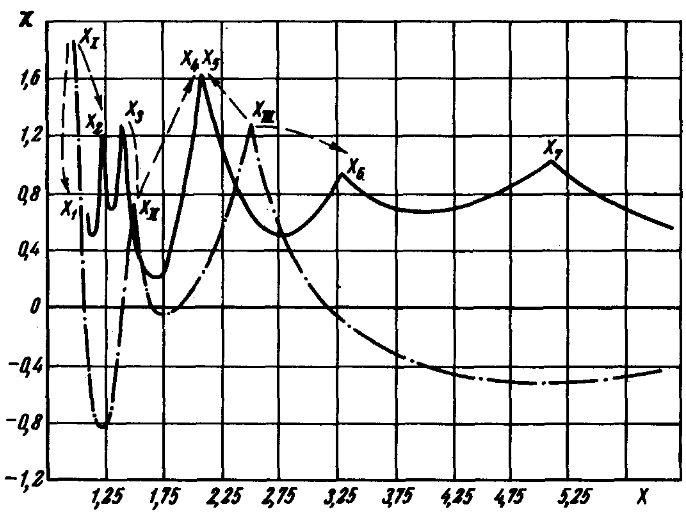
The effective-mass Hamiltonian for crystals with low symmetry (for example, with lattice of the Te type [1,2]) can have terms of the form $\vec{H}\cdot\vec{B}(\vec{k})$ [3], which are proportional to the magnetic field; it is necessary for this purpose that the wave-vector group admit of an invariant imaginary pseudo-vector $\vec{B}(\vec{k})$. The presence of such a term in the Hamiltonian gives rise to a relative displacement of the minima located at the points $\vec{k}$ and $-\vec{k}$ of the Brillouin zone, and leads to a certain singularity in all the oscillatory phenomena arising as a result of oscillations of the electron state density. By way of an example, we shall consider henceforth the de Haas - van Alphen effect.

1. The electron energy in a quantizing arbitrarily oriented magnetic field, for a




Oscillations of the magnetic susceptibility as a function of the dimensionless magnetic field:

$\eta_1 = (3 + b_1 + b_2)x_1,$
$\eta_2 = (3 + b_1 - b_2)x_2,$
$\eta_3 = (3 - b_1 + b_2)x_3,$
$\eta_4 = (3 - b_1 - b_2)x_4,$
$\eta_5 = (1 + b_1 + b_2)x_5,$
$\eta_6 = (1 + b_1 - b_2)x_6,$
$\eta_7 = (1 - b_1 + b_2)x_7,$
$\eta_I = (3 + b_1)x_I,$
$\eta_{II} = (3 - b_1)x_{II},$
$\eta_{III} = (1 + b_1)x_{III};$

the dash-dot curve denotes $\chi_1$ and the solid curve $\chi_2$.

valley having the symmetry of an ellipsoid of revolution, is

$$\epsilon(n, a, l, p_z) = \beta^* H \epsilon_{n,a,l} + \frac{p_z^2}{2m_2^*}, \quad \epsilon_{n,a,l} = 2n + 1 + ab_1 + lb_2, \tag{1}$$

$$a = \pm 1, \quad l = \pm 1, \quad \beta^* = e\hbar/2m_1^* c, \quad b_1 = g\beta_0/2\beta^*, \quad b_2 = B/\beta^*,$$

$m_1^*$, $m_2^*$, and B are the effective masses and the constant of the relative displacement of the minima; these displacements depend on the angle between the magnetic-field direction and the symmetry axis. The spin $b_1$ and the constant $b_2$ of the relative valley displacement enter in the energy in a perfectly symmetrical manner, and therefor for a nondegenerate electron gas the magnetic susceptibility $\chi$ contains two analogous paramagnetic terms:

$$\chi = \chi_0\left(\frac{1}{x} - \operatorname{cth} x + b_1 \operatorname{th} b_1 x + b_2 \operatorname{th} b_2 x\right)/x, \tag{2}$$

$$\chi_0 = N\beta^*/VkT, \quad x = \beta^* H/kT.$$

2. It is seen from (1) that the relative displacements of the minima should cause a two-fold splitting of the oscillatory peaks; the magnitude of this splitting will be illustrated by a numerical calculation for the de Haas - van Alphen effect.

Let us consider the case when T = 0 and only the levels with n = 0 and 1 are filled. The horizontal axis of the figure represents the magnetic field $H/H_0$, where $H_0$ is determined from the condition $\eta = 3 + b_1 + b_2$. The vertical axis represents $\chi = 1(1/x)(\partial\mathfrak{E}/\partial x)$. Here $\eta = \zeta/\beta^* H_0$ and $\mathfrak{E} = E/G$ are the dimensionless chemical potential and the electron gas energy, with

$$G = \frac{(2m_2^*)^{1/2} m_1^* V(\beta^* H_0)^{3/2}}{\pi^2 \hbar^3}.$$

The $\chi_1$ curve is plotted using the parameter values $b_1 = 0.6$ and $b_2 = 0$; the $\chi_2$ curve was plotted for $b_1 = 0.6$ and $b_2 = 0.4$. The two-fold splitting is clearly manifest even at the chosen small value of $b_2$. The arrows indicate the peaks of $\chi_1$ from which the peaks of $\chi_2$ are obtained.

3. The general case of arbitrary (but quantizing) fields and arbitrary temperatures can be analyzed by the method proposed in [4]. Calculations lead to formulas similar to the well known expressions for the de Haas - van Alphen effect, but with an additional paramagnetic term $\sim b_2^2$ and with an additional factor $\cos(r\pi b_2)$ in the oscillating sum.

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