STATE OF THE CONDENSON TYPE IN A STRONG MAGNETIC FIELD

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Deigen and Pekar [1] advanced the hypothesis that the conduction electron in a homopolar crystal is capable of deforming the lattice and of becoming localized in the region of the deformation produced by it in such a manner that the energy of this self-consistent state is smaller than the electron energy in the undeformed lattice. The authors called such a state a condenson, and showed in the same article that no condensons of large radius are realized. One can expect, however, condenson states to be produced by an external magnetic field, since any arbitrarily shallow potential well leads in a strong magnetic field to the appearance of local electron states.* We present below an attempt at the corresponding calculation.

Let us consider a conduction electron in a cubic homopolar crystal in the presence of a strong magnetic field. Since all the characteristic dimensions of the problem are much larger than the lattice constant, the lattice can be replaced by an elastic continuum with a deformation tensor u_{ij} and with elastic moduli $\lambda_{ik\ell m}$.

To describe the interaction between the electron and the lattice, we shall use the method of the deformation potential. The ground state of the system will be sought by a variational method (see [1]). By minimizing with respect to u_{ij}, we reduce the problem to a determination of ψ_0 - the normalized extremal of the functional

$$I(\psi) = \frac{1}{2m}\int \psi^*(\mathbf{p} + \frac{e}{c}\mathbf{A})^2\psi\, d\tau - \frac{3\epsilon_1^2}{2(\lambda_{1111} + 2\lambda_{1122})}\int|\psi|^4 d\tau, \tag{1}$$

where ϵ_1 is the constant of the deformation potential. Let us ascertain the class of functions in which ψ_0 should be sought. To this end we note that the assumed approach is equivalent to the adiabatic approximation, and ψ_0 is the electron wave function of the ground state, calculated for u_{ij} corresponding to the sought self-consistent state, i.e.,

$$\left[\frac{1}{2m}(\mathbf{p} + \frac{e}{c}\mathbf{A})^2 + V_0(\mathbf{r})\right]\psi_0 = E_0\psi_0; \tag{2}$$

$V_0(\vec{r})$ is a shallow potential well having axial symmetry and an axis along $\vec{H}$. Going over to cylindrical coordinates, we can easily show that when the magnetic field is sufficiently strong, so that it is sufficient to retain only the first Landau band in the expression of ψ_0 in terms of the states of the free electron in the magnetic field (see [2]), the ground state (2) takes the form

$$\psi_0(\mathbf{r}) = \frac{1}{\sqrt{2\pi}\rho_0}\exp\left\{-\frac{\rho^2}{4\rho_0^2}\right\}\chi(z), \tag{3}$$

where $\rho_0 = \sqrt{c\hbar/eH}$ is the characteristic magnetic length, i.e., it corresponds to zero value of the quantum number m. Substitution of (3) in (1) leads to a variational problem whose Euler-Lagrange equation is of the form

$$-\frac{\hbar^2}{2m}\frac{d^2\chi}{dz^2} - \frac{3}{2}\frac{\epsilon_1^2}{\lambda_{1111} + 2\lambda_{1122}}\frac{1}{2\pi\rho_0^2}\chi^3 = E^*\chi \qquad (4)$$

with $E^* = E - \mu H$ (μ - Bohr magneton). The ground state of (4), satisfying the normalization condition, is

$$\chi_0(z) = \pm\left(\sqrt{2r_z}\,\mathrm{ch}\,\frac{z - z_0}{r_z}\right)^{-1}, \qquad (5)$$

where z_0 is an arbitrary constant and the energy of the local state and its radius are

$$E_0^* = -\frac{\hbar^2}{2mr_z^2}, \qquad (6)$$

$$r_z = \frac{\hbar^2}{2m}\frac{16\pi\rho_0^2(\lambda_{1111} + 2\lambda_{1122})}{3\epsilon_1^2}, \qquad (7)$$

i.e., $r_z \sim H^{-1}$ and $E_0^* \sim H^2$. The net gain in energy due to the formation of the self-consistent state (the intrinsic energy of the condenson) is $(1/3)|E_0^*|$. For a numerical estimate we assume $m = m_0$, $\epsilon_1 = 10$ eV, $\lambda_{1111} + 2\lambda_{1122} = 10^{11}$ erg/cm^3, and $H = 5 \times 10^5$ Oe. Then $r_z = 0.6 \times 10^{-6}$ cm and $E_0^* = 1.4 \times 10^{-3}$ eV.

If the effective mass of the "bare" electron is anisotropic, then the optimal conditions for the appearance of the states under consideration occur when the magnetic field is directed along the largest axis of the effective-mass ellipsoid. It is important to note that allowance for the next Landau bands in the expansion of ψ_0 does not lead to violation of the condenson states, although it does complicate the problem.

Owing to translational symmetry of the problem, the condenson is not a static formation, but has a finite effective mass and dispersion, although the calculation of these quantities, as well as allowance for the quantization of the lattice vibration, is beyond the scope of the present communication. I take this opportunity to thank E. I. Rashba for discussing the results and for valuable remarks.

[1] M. F. Deigen and S. I. Pekar, Zh. Eksp. Teor. Fiz. 21, 803 (1951).
[2] E. I. Rashba, Opt. Spektrosk. 2, 88 (1957).

* A seemingly analogous situation occurs in one-dimensional molecular structures where, as shown by Rashba [2], the exciton-phonon interaction gives rise to excitations of the exciton type, which cause local deformation of the chain and have a finite radius.