The measured maximum temperature of the helium amounted to 200 eV. The doubling of the temperature on doubling the magnetic field agrees with formula (5).

3. To determine the rate of heating it is necessary to know the noise energy density. It seems obvious that the amplitude of the electron-velocity oscillations $v_\sim$ cannot exceed the phase velocity of the oscillations $\omega/k \simeq u$. The energy density of the oscillatory motion of the electrons $nmv_\sim^2/2$, can therefore be assumed to be the limiting noise energy w. If we substitute this estimate in (4), we obtain for the ion heating rate the expression

$$n\dot{T}_i = \gamma_i w \lesssim (\omega_{Hi}\,\omega_{He})^{1/2}\frac{nmu^2}{2}. \tag{6}$$

We chose for γ_i the maximum possible increment. We can now estimate the damping decrement of the electromagnetic wave $\delta = 4\pi P_i/H_\sim^2$. For a forward magnetosonic wave:

$$u = \frac{2\pi f}{\omega_{Hi}}\frac{H_\sim}{\sqrt{4\pi n M}}, \quad \delta = \frac{(2\pi f)^2}{(\omega_{Hi}\,\omega_{He})^{1/2}}. \tag{7}$$

The value of δ calculated from (7) agrees with the experimentally measured value.

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* The measured temperature of the ions in the center of the heating circuit are given in [1].

"DISSIPATIVE" INSTABILITY OF A LIGHT WAVE IN A NONLINEAR DIELECTRIC

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It is well known [1-3] that a plane wave in a nonlinear self-focusing medium is unstable. We shall show in this paper that if account is taken of the finite relaxation time of the nonlinearity, the wave turns out to be unstable also in a defocusing medium. In addition, introduction of a finite relaxation time changes strongly the character of the instability of a plane wave in a self-focusing medium.

We write down the equations for the eikonal Φ and the amplitude A in the form (see [3])

$$\left(\frac{\partial}{\partial t} + v_{gr}\frac{\partial}{\partial x}\right)\Phi + \frac{\omega_k''}{2}\left(\frac{\partial\Phi}{\partial x}\right)^2 + \frac{v_{gr}}{2k_0}(\nabla_\perp\Phi)^2 =$$

$$= -qp + \frac{1}{2A}\left(\omega_k'' \frac{\partial^2 A}{\partial x^2} + \frac{v_{gr}}{k_0}\Delta_\perp A\right),$$

$$\left(\frac{\partial}{\partial t} + v_{gr}\frac{\partial}{\partial x}\right)A^2 + \omega_k'' \frac{\partial}{\partial x} A^2 \frac{\partial \Phi}{\partial x} + \frac{v_{gr}}{k_0}(\nabla_\perp A^2 \nabla_\perp \Phi) = 0. \quad (1)$$

Here $Q = -\omega^2\epsilon_2/(\omega^2\epsilon\omega)'$, p is the polarization, which satisfies the relaxation equation (see [4])

$$-L\frac{\partial p}{\partial x} + p = A^2, \quad L = \tau v_{gr},$$

and τ is the characteristic relaxation time. For the Kerr nonlinearity mechanism, the value of L is of the order of 1 mm.

Let us linearize Eqs. (1) against the background of a plane wave with amplitude A_2 and assume that the perturbations of all the quantities are proportional to the factor $\exp[-i\Omega t + i(kr)]$; we obtain the dispersion equation

$$(\Omega - k_x v_{gr})^2 = (\omega_k'')^2 \xi^2\left[\frac{1}{4}\xi^2 \pm \frac{k_0^2}{1 - ik_x L}\right],$$
$$\xi^2 = k_x^2 + \frac{v_{gr}}{k_0 \omega_k''}k_\perp^2, \quad k_0^2 = \frac{|q|A_0^2}{\omega_k''}, \quad (2)$$

k_0 is the characteristic reciprocal dimension of the self-focused beam with amplitude A_0. Positive and negative signs in (2) correspond to defocusing and self-focusing media, respectively. The presence of an imaginary part in (2) indicates the existence of the instability regardless of the value of k or the sign of q.

The characteristic parameter of the problem is the quantity k_0L. Let us consider the case $k_0L \ll 1$. In a defocusing medium, the maximum of the increment lies in this case in the region $k \sim 1/L$ and its order of magnitude is $\gamma_{max} \sim qA_0^2$, i.e., the same order of magnitude as the shift of a plane wave as a result of the nonlinearity. In a self-focusing medium, the instability increment has two maxima. The first lies in the region $k \sim k_0$ - this is the ordinary instability of the plane wave [1,2], and the finite character of the relaxation affects it little. The second maximum is in the region $k \sim 1/L$, where the instability has the same character as for a defocusing medium. In both maxima, the order of the increment is $\gamma \sim qA_0^2$.

In the case $k_0L \gg 1$ the self-focusing and defocusing media behave in similar fashion. The maximum increment lies in the region $k_x \sim 1/L$, $k \sim k_0$ and is of the same order, $\gamma \sim qA_0^2$.

The appearance of additional instabilities when allowance is made for the finite relaxation time can be explained in the following manner. When $\tau = 0$ Eqs. (1) are equations of the hydrodynamic type; their integrals of motion are

$$M = \int A^2 \, d\mathbf{r}, \quad \mathbf{p} = \int A^2 \nabla\Phi \, d\mathbf{r},$$

$$\mathcal{E} = \frac{1}{2}\int\left\{A^2\left[\omega''_k\left(\frac{\partial\Phi}{\partial x}\right)^2 + \frac{v_{gr}}{k_0}(\nabla_\perp\Phi)^2\right] + \left[\omega''_k\left(\frac{\partial A}{\partial x}\right)^2 + \frac{v_\perp}{k_0}(\nabla_\perp A)^2\right] + \frac{q}{2}A^4\right\} d\mathbf{r}.$$

These integrals can be identified with the hydrodynamic "mass," "momentum," and "energy" integrals. Allowance for the finite relaxation time makes the "momentum" and the "energy" quantities that are no longer conserved, so that allowance for the finite relaxation time is equivalent to introducing some dissipation mechanism, which incidentally conserves the true energy of the light field - the "mass" integral. It follows therefore that the instability obtained above can be set in correspondence with the "dissipative" hydrodynamic instabilities that arise when account is taken [5,6] of small dissipative terms (say viscosity). Just as in our case, the maximum increment of these instabilities may be independent of the magnitude of the dissipation [6].

Let us see the consequences to which "dissipative" instability can lead. When $k_0L \ll 1$ the real part of the frequency Ω in the region of the maximum increment is much larger than the imaginary part, and we can expect a "weak turbulence" of the modulation wave, with characteristic length on the order of L, to develop against the background of the plane wave; the phases of these waves become randomized. In final analysis this should lead to broadening of the line to a value $\Delta\omega \sim \omega(1/kL)$ ($2\pi/L$ is the wavelength of the light), and also to the appearance of additional collective dissipative effects for the motions, with dimension on the order of $1/k_0$.

If $k_0L \gg 1$ the real and imaginary parts of the frequency in the region of the maximum increment are of the same order. At the same time, strong turbulence of the light field takes place and leads, during the initial stage, to the formation of light bunches with longitudinal dimension L and transverse dimension $1/k_0$. Allowance for the finite dissipation also should cause the self-focused beam to break up into segments of length on the order of L.

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