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INTERACTION OF VORTEX FILAMENTS IN SUPERCONDUCTORS OF THE SECOND KIND WITH THE ELASTIC DEFORMATION FIELD

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The large critical currents (10^5 - 10^6 A/cm^2) attained presently in superconductors of the second kind with large κ [1] are attributed [2] to the presence of various crystal-lattice defects or other inhomogeneities in such superconductors. These hinder the viscous motion of the vortex filaments in the mixed state [3] under the action of the Lorentz force produced by the transport current. Such "blocks" to the vortex filaments may be, in particular, dislocations (see [4] and the bibliography in the review [5]) and other elastic inhomogeneities.

The interaction of vortices with the elastic field is determined by two principal factors. In the mixed state of a deformed superconductor, owing to the difference between the elastic moduli of the superconducting and normal states, an additional elastic energy [4] (quadratic in the deformations) is produced as a result of the alternation of the normal regions in the vortex lattice. At the same time, owing to the contribution of the magnetic and kinetic energies of the superconducting condensate to the pressure, there is also a magnetoelastic interaction which is linear in the deformations. At atomic distances near the dislocation line, both contributions to the energy are in general comparable in magnitude. Actually, however, owing to the macroscopic coherence of the electronic states, which is characteristic of superconductors, the elastic dislocation field is effectively smoothed out over distances on the order of the electron correlation radius, which greatly exceeds the atomic radius of the dislocation core. The main role should therefore be played by the magnetoelastic interaction that is linear in the deformation, whereas the quadratic terms should be negligibly small.*

To calculate this interaction it is sufficient to determine the change in the energy of the vortex lattice under the influence of the elastic field. It is known [3] that at large κ and not too strong magnetic fields, $H \ll H_{c2}$, the vortices can be described (see [3,6]) by the London local equations [7]:

$$\mathrm{curl}\,\mathbf{H} = 4\pi N_s e \mathbf{v}_s\ ; \mathrm{curl}\,\mathbf{v}_s = -(e/m)\,\mathbf{H}, \tag{1}$$

where $\vec{v}_s = (\nabla\chi - 2\vec{e}\cdot\vec{A})rm$ is the velocity of the superconducting condensate ($\hbar = c = 1$), χ is the phase of the ordering parameter, $\vec{H} = \mathrm{curl}\,\vec{A}$, and N_s is the density of the superconducting electrons. In the same approximation, the energy is equal to

$$E = \int dV \left[\frac{H^2}{8\pi} + \frac{N_s m v_s^2}{2} \right]. \quad (2)$$

The presence of an elastic field leads to a spatial change in the density $N_s(r)$; in the simplest isotropic case this change can be described locally as follows:

$$\frac{N_\delta(\mathbf{r})}{N_s} = 1 + \gamma u_{ll}(\mathbf{r}); \quad \gamma = V \frac{\partial}{\partial V} (\ln N_s)_T \quad (3)$$

(u_{ll} is the trace of the strain tensor). When T = 0 and in a contaminated superconducting alloy ($l \ll \xi_0$, l - mean free path, $\xi_0 \sim v_0/T_c$ - correlation radius in a pure superconductor), N_s is determined by the following formulas [8]:

$$N_s = \pi N \tau_{tr} \Delta(0); \quad \frac{1}{\tau_{tr}} = \frac{n m p_0}{\pi} \int \frac{do}{4\pi} |u(\theta)|^2 (1 - \cos\theta),$$

where N and n are the densities of the electrons and of the impurities, $\Delta(0) \sim T_c$ is the BCS gap at T = 0, and τ_{tr} is the transport free path time. From this we can easily get the value of γ [3]:**

$$\gamma = \frac{1}{3} + V \frac{\partial}{\partial V} \ln T_c . \quad (4)$$

The vortex lines are defined mathematically as the singularity lines of the superconducting velocity v_s, such that the integral over an infinitesimally small contour C_i enclosing the filament is equal to [3]

$$\oint_{C_i} v_s \, d\mathbf{r} = \frac{\pi}{m} .$$

Taking this condition into account and integrating the first term in (2) by parts with the aid of (1), we obtain [3]

$$E = \frac{1}{8e} \sum_i \int dl_i H(\mathbf{r}_i), \quad (5)$$

where the integration is along the filament and the sum is taken over all the filaments. Similarly, integration of the first equation of (1) over an infinitesimally small contour enclosing the filament yields

$$-\oint_{C_i} \frac{\partial H}{\partial \rho} \rho \, d\phi = \frac{\pi}{e\delta^2} \frac{N_s(\mathbf{r}_i)}{N_S}; \quad \frac{1}{\delta^2} = \frac{4\pi N_S l^2}{m}$$

(δ is the depth of penetration of the field). From this, with allowance for (3), it follows that in the vicinity of the vortex filament we have

$$H \simeq \frac{1 + \gamma u_{ll}(\mathbf{r}_i)}{2e\delta^2} \ln \frac{\delta}{\rho_i} + H_{reg}, \tag{6}$$

where ρ_i is the distance from the filament.

When (6) is substituted in expression (5) for the energy, the regular term H_{reg} determines the energy of interaction of the vortex filament, and the first term after cutoff at the radius of the vortex core, $\xi \sim \sqrt{l\xi_0}$, gives a logarithmically large self energy of the vortices [3], which includes in our case the potential U of the interaction between the vortices and the elastic field ($E = \epsilon_0 \sum_i \int dl_i + U$). Thus the potential energy of vortex filaments in an external elastic field assumes finally, with logarithmic accuracy, the form

$$U = \gamma\epsilon_0 \sum_i \int dl_i\, u_{ll}(\mathbf{r}_i), \quad \left(\epsilon_0 = \frac{\ln\kappa}{16(e\delta)^2}\kappa\, \frac{\delta}{\xi}\right), \tag{7}$$

where ϵ_0 is the self energy of the vortex per unit length and γ is given by (3) and (4).

By varying (7), we can readily determine the force acting on a unit length of the filament in the elastic field:

$$\mathbf{F} = -\frac{\delta U}{\delta \mathbf{r}} = \gamma\epsilon_0 [\tau[\tau \nabla u_{ll}]] + \frac{\mathbf{n}}{R} u_{ll}].$$

Here τ and $\mathbf{n}$ are the unit vectors tangential and normal to the vortex filament and R is the radius of curvature of the filament.

By way of an example, let us consider the interaction of a straight vortex filament situated in the (x, y) plane and parallel to the x axis, with a wall of edge dislocations directed along the z axis and lying on the x axis parallel to the Burgers vector b. In this case u_{ll} (see [9]) and the filament potential (7) per unit length (U = Lu, L - length of filament) take, as can be easily verified, the form

$$u_{ll}(x,y) = \frac{1/2 - \sigma}{1 - \sigma} \frac{b}{d} \frac{\mathrm{sh}^2(\pi y/d)}{\mathrm{ch}^2(\pi y/d) - \cos^2(\pi x/d)};$$
$$u = \frac{1/2 - \sigma}{1 - \sigma} \frac{\gamma\epsilon_0 b}{d} \operatorname{sign} y \tag{8}$$

(d is the distance between dislocations). It follows from the foregoing that the potential jump (8) at $\gamma = 0$ should be "smeared" over a distance on the order of ξ.*** The maximum force F_c per unit vortex length is therefore of the order of

$$F_c \approx \frac{\gamma\epsilon_0 b}{d\xi} \frac{1/2 - \sigma}{1 - \sigma} \sim \gamma \ln\kappa H_c^2 b \frac{\xi}{d} \frac{1/2 - \sigma}{1 - \sigma}.$$

The corresponding critical current j_c is determined from the equality of the Lorentz force $c^{-1}(j\Phi_0)$ ($\Phi_0 = \pi\hbar c/e$ - quantum magnetic flux of vortex) to the force F_c. The maximum value of j_c is obviously attained when $d \sim \xi$. Estimates show that values $j_{c_{max}} \sim 10^4 - 10^6$ A/cm^2 are attainable at a dislocation density $10^{10} - 10^{12}$ cm^{-2}, giving sensible orders of magnitude.

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* This circumstance is not taken into account in [4,5], where the linear terms are discarded.

** For most superconductors, γ is apparently determined essentially by the derivative $\partial T_c/\partial V$. For tin, for example, $\gamma \sim 10$.

*** This question calls for further research, since u_{ll} in contaminated alloys is smoothed out over the mean free path l. In the assumed approximation, however, the distance between the vortex filament and the dislocation is determined accurate to the radius of the vortex core $\xi \sim \sqrt{\xi_0/l} \gg 1$. By way of precaution we assume here a low estimate of the current j_c, corresponding to the smoothing of the jump (8) over the length ξ.