ADMISSIBLE MASS DISTRIBUTIONS OF UNSTABLE PARTICLES

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In [1] we introduced the concept of mutual admissibility of the mass distributions of associatively created unstable particles, and obtained the necessary conditions for the admissibility of mass distributions specified by their analytic structure. We present below the necessary and sufficient conditions for admissibility of mass distributions of unstable particles in the most general formulation.

1. We consider a quasi-two-particle reaction:

$$m_1 + \dots + m_i + \dots + m_{N_1} \to m + M \to m_{N_1+1} + \dots + m_k + \dots + m_{N_2} + m_{N_2+1} + \dots + m_\ell + \dots + m_{N_3}, \tag{1}$$

where $N_1 \geq 2$, $N_2 - N_1 \geq 2$, and $N_3 - N_2 \geq 2$, i.e., an arbitrary $N_1 + N_2 + N_3 = N \geq 6$-point diagram. In (1), all the numbered particles are stable (absolutely), and m and M are unstable particles (resonances) or simply groups of particles

$$m \equiv \sum_{k=N_1+1}^{N_2} m_k \text{ and } M \equiv \sum_{\ell=N_2+1}^{N_3} m_\ell$$

in the direct reaction corresponding to (1). If the energy-momentum conservation law "from stable to stable" [2,3] holds in reaction (1), then the masses m and M are related as follows [1]:

$$M^2 = m^2 + (E_0^2 - p_0^2) - 2E_0\sqrt{m^2 + p_m^2} + 2(p_0, p_m), \tag{2}$$

where

$$E_0 \equiv \sum_{i=1}^{N_1} \sqrt{m_i^2 + p_i^2},\ p_0 \equiv \sum_{i=1}^{N_1} p_i,\ E_m \equiv \sum_{k=N_1+1}^{N_2} \sqrt{m_k^2 + p_k^2},\ p_m \equiv \sum_{k=N_1+1}^{N_2} p_k,$$

$$E_M \equiv \sum_{\ell=N_2+1}^{N_3} \sqrt{m_\ell^2 + p_\ell^2},\ p_M \equiv \sum_{\ell=N_2+1}^{N_3} p_\ell,\ m^2 \equiv E_m^2 - p_m^2,\ M^2 \equiv E_M^2 - p_M^2.$$

If we assume for simplicity at first that $(\vec{p}_0 \cdot \vec{p}_m) = p_0 p_m$, then (2) leads [1,4] to rigorous relations between the densities of the conditional mass distributions $\omega(m|p_m)$ and $W(M|p_m)$ at fixed momenta P_m and p_M, and between the momentum distribution densities $\bar{\omega}(p_m)$ and $\bar{W}(p_M)$:

$$\begin{cases} \omega(m|p_m)dm = W(M|p_M)dM, \\ \bar{\omega}(p_m)dp_m = \bar{W}(p_M)dp_M. \end{cases} \tag{3}$$

Accordingly, for the unconditional distribution of the masses $\omega(m)$ and $W(M)$, which are directly connected with the probabilities of the corresponding physical processes, we obtain

$$\begin{cases} \omega(m) = \int \bar{\omega}(p_m)\,\omega(m|p_m)\,dp_m, \\ W(M) = \int \bar{W}(p_M)\,W(M|p_M)\,dp_M. \end{cases} \tag{4}$$

In the most general case we have for the reaction (1)

$$\begin{cases} \omega(m) = \int \ldots \int |S(m, M, q_a)|^2\, dM \prod_a dq_a, \\ W(M) = \int \ldots \int |S(m, M, q_a)|^2\, dm \prod_a dq_a, \end{cases} \tag{5}$$

where $S(m, M, q_\alpha)$ is the S-matrix of the reaction (1) and includes the corresponding δ-functions ensuring the energy-momentum conservation law "from stable to stable" [2,3], and q_α are the relativistic invariants, other than m and M, on which the S-matrix of the reaction (1) can depend. It is precisely these distributions, $\omega(m)$ and $W(M)$, which are being investigated in the study of resonances. It is perfectly obvious that when E_0, $|p_0| < \infty$ the function $\psi(m, M)$, defined in accordance with

$$\psi(m, M) \equiv \int_a \ldots \int |S(m, M, q_a)|^2 \prod_a dq_a \tag{6}$$

differs from zero by virtue of the energy-momentum conservation law, only in a closed set G of the plane $R^2(m, M)$, and

$$\iint_G \psi(m, M)\, dm\, dM = 1 \tag{7}$$

As noted by the author and by V. N. Sudakov, it is possible to derive from Sudakov's recent mathematical results concerning measurable decompositions [4] the following fundamental theorem:

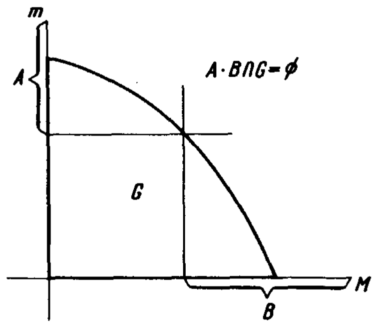
Theorem on admissible mass distributions: Let G be a closed set of the plane $R^2(m, M)$ (see the figure) with unity measure [Eq. (7)]. The necessary and sufficient condition for the admissibility of the mass distributions $\omega(m)$ and $W(M)$ is the satisfaction, for an arbitrary breakdown of the axes m and M into measurable subsets A and B such that $A \times B \cap G = \emptyset$ (empty) (see the figure), of the following condition:

$$\int_A \omega(m)\, dm + \int_B W(M)\, dM \leqslant 1 \tag{8}$$

2. As a consequence of the fundamental theorem we obtain a result similar to that [1]:
Theorem: The mass distributions

$$\begin{cases} \omega(m) = A_m[(m - m_0)^2 + \Gamma_m^2]^{-1}, \\ \qquad\qquad (m, M), (m_0, M_0) \in G, \\ W(M) = A_M[(M - M_0)^2 + \Gamma_M^2]^{-1}, \end{cases} \tag{9}$$

where A_m and A_M are normalization constants, mutually indmissible for arbitrary m_0, Γ_m, M_0, and Γ_M (and all the more for $\Gamma_m \lessgtr \Gamma_M$).



3. Let us consider the main physical consequences. For ordinary unstable particles, owing to the existence of a reaction of the type $\gamma + p \to n + \pi^+$, we arrive by virtue of the theorems of the present paper at the following alternatives: a) the mass distributions do not admit of continuation to the complex plane and as a consequence the decays of unstable particles, particularly of the neutron and the positive pion, should be essentially non-exponential; b) on the other hand, if a careful experiment (see [5] in this connection) shows the decay to be exponential, this means violation of the energy-momentum conservation in reactions in which unstable particles are produced, accurate to within the decay widths. In pair production of resonances, inasmuch as their widths are of the same order, the pole distributions (the Breit-Wigner formulas) are mutually admissible. Exceptions are resonance pair-production reactions in which one resonance is the η meson. Resonances with the same discrete quantum numbers, produced simultaneously with ordinary unstable particles, can no longer be described by simple pole distributions, thus indicating clearly that the resonance mass distribution is dependent on the preparation [2,3]. In this connection, interest attaches to data on the dependence of the distribution of the ρ-meson masses on the preparation [2,5-7]. When account is taken of crossing symmetry, the inadmissibility of pole distributions of the masses for the resonances obviously leaves no hope for the validity of the Regge-pole method.

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SPATIAL SYNCHRONISM IN NONSTATIONARY PROCESSES OF THE "PHOTON-ECHO" TYPE

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This article describes several effects that exhibit a number of singularities compared with the well-known "photon-echo" effect, which was recently predicted [1] and investigated