

Substituting here (7), we get

$$B(0) = \frac{10}{3} \frac{z^2 e^2}{\Omega_0^{4/3}(0)} \xi. \quad (9)$$

We note that the small term omitted from (8), corresponding to the electronic compressibility, has (like $P^{(0)}$) a dependence on Ω_0 lying between $1/\Omega_0^{5/3}$ and $1/\Omega_0^{4/3}$, as follows directly from the usual expression for the energy of the electron gas [8]. We can therefore write with good approximation

$$B(0) \sim \frac{1}{\Omega_0^{4/3}(0)}$$

Hence, returning to the general expression for $P^{(3)}$ and taking (7) into account, we get

$$\frac{P}{B(0)} \approx f \frac{\Omega_0}{\Omega_0(0)}, \quad (10)$$

which is an approximate universal relation for alkali metals. A similar relation was obtained empirically by analyzing the measurements for the entire group of alkali metals [1]. We emphasize that such a simple analysis would not be valid in the general case for an arbitrary metal, owing to the appreciable role of P^2 , and also of the many-particle terms in (3).

The figure shows a theoretical curve for the complete equation of state (with allowance for $P^{(2)}$ and $P^{(0)}$ at $T = 0$), in terms of the variables $P/B(0)$ and $\Omega_0/\Omega_0(0)$. The figure shows also the experimental points obtained for Na [2] and K [3]. We see that the agreement between theory and experiment is good in a wide range of applied pressures.

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MAGNETIC SURFACE LEVELS IN SUPERCONDUCTORS

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Submitted 12 May 1969

ZhETF Pis. Red. 10, No. 1, 49 - 53 (5 July 1969)

Pinkus [1] has recently pointed out the possibility, in principle, of the existence in superconductors of single-particle bound states due to the penetration of the magnetic field. The physical cause of the appearance of discrete levels is the quantization of the finite motion of the quasiparticles in the potential well produced by the magnetic field near the

surface of the metal. We determine in the present communication the spectrum of the single-particle excitations in a superconductor in the quasiclassical approximation, and present a simple physical interpretation of the quantum states.

We start from Gor'kov's equations for the wave functions of the quasiparticles [2]. The superconductor occupies a half-space $x > 0$. The vector potential is $A = (0, -\int_x^\infty H(x')dx', 0)$. We assume on the superconductor boundary the condition of specular reflection, which takes in the quasiclassical approximation the form

$$g|_{x=0} = f^+|_{x=0} = 0. \quad (1)$$

(We note that the form of the boundary conditions in the absence of diffuse scattering has no bearing on the determination of the spectrum). Almost specular reflection is due to small angles ϕ of collision between the almost-glancing electrons of interest to us and the surface, $\phi \approx \phi_0 = (\delta/r)^{1/2} \ll 1$ (r - cyclotron resonance, δ - depth of penetration of magnetic field). Inasmuch as the number of such electrons is small ($\sim \phi_0$), the energy gap Δ is determined, as before, by the self-consistent interactions inside the volume of the metal. The gap is constant in the approximation linear in H .

We seek the wave functions in the form

$$\psi(r) = \begin{pmatrix} A \\ B \end{pmatrix} \exp(i P_y y + i p_z z) \exp(i S(x)), \quad (2)$$

where in the quasiclassical approach $S(x)$ is a rapidly varying function of the coordinate. The condition for the validity of the quasiclassical approach has in the typical case the form $H > (a/\delta\kappa)H_c$, where a is of the order of the interatomic distance, H_c is the critical field, and κ is the Ginzburg-Landau parameter. The generalized momentum of the system $p = \partial S/\partial x$ has in the main approximation in the quasiclassical parameters is given by

$$p_\pm^2/2m = \mu - \epsilon_\pm \pm [(\epsilon + \frac{e}{mc} A_y P_y)^2 - \Delta^2]^{1/2}, \quad (3)$$

where $\epsilon_\pm = (p_z^2 + p_y^2)/2m$, and the remaining notation is that of [2]. A plot of the functions $p_\pm(x)$ at the most interesting values of the parameters is shown in Fig. 1 (only $\mu - \epsilon_\pm = \alpha > 0$ are considered, since $p_-^2(-\alpha) = -p_+^2(\alpha)$). It is important that, as can be seen from (3), the velocity $v_\pm = \partial \epsilon/\partial p_\pm$ vanishes at the point of "termination" of the real branches (i.e., on the boundary of the classically accessible region). The $p_\pm(x)$ dependence corresponds to the classical trajectory $v_\pm(x) = \dot{x}_\pm(x)$ shown in the same figure. Obviously, discrete quantum levels correspond to periodic trajectories near the metal surface (see Fig. 1).

Leaving the detailed analysis of the possible situation to a more comprehensive article, we confine ourselves here to the characteristic case corresponding to Fig. 1. In this case the only turning point x_0 , common to the electron and "hole" (we call electron and hole, arbitrarily, the branches distinguished by the indices "+" and "-"), is the point of reflection for the particles. An electron traveling from the surface arrives at the point x_0 with momentum p_+ , is reflected at this point, and is transformed into a "hole" with momentum p_- (it is "forbidden" to go off with a different momentum by the presence of a momentum jump between the pairs of branches at $x = x_0$). Accordingly "hole" ($-p_-$) arriving at the point x_0

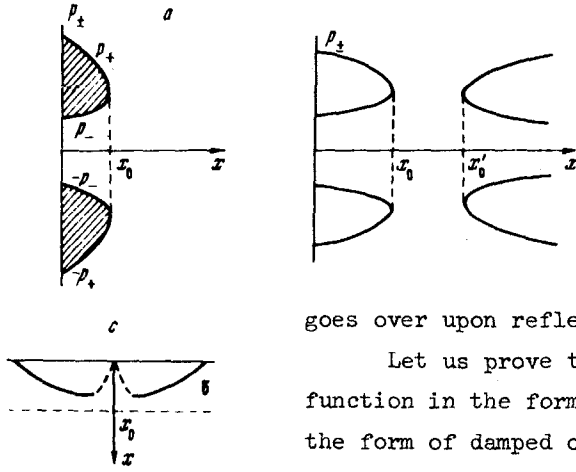


Fig. 1. Phase trajectories of excitations
 $a > [(-\frac{e}{mc} AP_y)^2 - \Delta^2]^{1/2}$: $a - P_y < 0$, $b - P_y > 0$,
 c - trajectory in real-space

goes over upon reflection into an electron with momentum $(-p_+)$.

Let us prove this result. As usual, we represent the wave function in the form of oscillating exponentials when $x < x_0$ and in the form of damped ones when $x > x_0$. The amplitudes B at $x > x_0$ are connected with the amplitudes A at $x < x_0$ by a transition matrix

$$\hat{\beta} = \begin{pmatrix} b & 0 \\ 0 & b^* \end{pmatrix}, \quad (4)$$

where the elements of the 2×2 matrix \hat{b} are of the order of unity. The fact that the amplitudes A and B corresponding to different pairs of branches do not become interconnected is the consequence of the presence of a momentum jump at the point x_0 . This can be shown formally by reducing the system of Gor'kov's equations near the turning point to an equation with linear coefficients, and then integrating this equation by the Laplace method (cf., e.g., [3]), making it possible simultaneously to find the exact solution for the excitations moving at the very surface in a practically homogeneous field. The fact that the 2×2 matrices \hat{b} are complex conjugate can be readily established from symmetry considerations.

From the condition that the amplitudes corresponding to solutions that grow inside the metal must vanish, we obtain by using the boundary conditions (1), accurate apart from the phase γ , the condition for the quasiclassical quantization of the single-particle excitations in the superconductor

$$S = \int_{x_0}^{x_0} (p_+ - p_-) dx = (n + \gamma) \pi. \quad (5)$$

Assume for concreteness that the field attenuates exponentially on penetrating the metal, $H = H_0 \exp(-x/\delta)$. Then the area S shown shaded in Fig. 1a can be expressed in terms of elliptic integrals. Without presenting here the concrete calculations, we give by way of an example one particular case of formula (5), which can be used for estimates. When

$$(\mu - \epsilon_1)/\Delta \gg [(\epsilon - \Delta + \Omega\delta |P_y|)/\Delta]^{1/2} \quad (6)$$

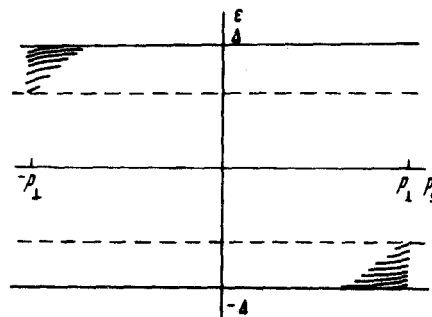
$$\epsilon = \Delta - \Omega\delta |P_y| + \Omega |P_y| \left[\frac{3}{4}(n + \gamma) \pi \hbar \right]^{2/3} \left(\frac{\mu - \epsilon_1}{m\Omega\Delta |P_y|} \right)^{1/3},$$

where $\Omega = eH_0/mc$.

At reasonable values of the parameters, several levels may fit within the depth δ ; with further change of P_y , these levels give way to a continuous spectrum. They are shown schematically in Fig. 2.

A similar analysis can be made also for all the remaining cases. Its result, however, is clear already from Fig. 1. In all cases, the quantization corresponds to a classical

Fig. 2. Excitation spectrum near surface



periodic motion, the distance between the levels $\Delta\epsilon$ being determined by its period T , vis., $\Delta\epsilon = n/T$. For example, it is clear that in the case of Fig. 1c the levels split into a band whose width is determined by the probability of tunneling of the quasiparticles from one region of classically-allowed motion to another, and decreases exponentially with increasing distance between these regions. It follows also from (3) that even a weak magnetic field decreases the energy gap in the spectrum of the quasiparticles near the surface, and moreover can cause it to vanish. In fact, at $H = 0$ the spectrum becomes continuous, and all energies compatible with the classical motion are allowed. It is seen from (3) that when $H = 0$ this leads to $|\epsilon| > \Delta$, whereas even $\epsilon = 0$ is admissible when $H \neq 0$, and $|P_y| = \Delta/\Omega\delta$. Since $P_y \approx P_0$, we must have $\Omega\delta p_0 \geq \Delta$ for the gap to vanish. For typical metals a shift of order Δ takes place in fields $H \approx 10$ Oe. Of course, the corresponding field must be smaller than the critical field. In the London case the corresponding field is of the order of the thermodynamic critical field, and in the Pippard case it may be appreciably smaller. It must be borne in mind, however, that the number of states of such quasiparticles is small and tends to zero as $H \rightarrow 0$.

In conclusion, let us consider normal excitations for which $\epsilon \gg \Delta$. The trajectory is periodic when $P_y < 0$ and $|P_y| > p_-$. In the opposite case, the electron leaves the skin layer. The period of motion and accordingly the distance between the energy levels of the normal electron is determined in the former case by the formula

$$T = \frac{m\delta}{(P_y^2 - p_-^2)^{1/2}} \left(\pi + 2 \arcsin \frac{p_-^2 - P_y^2 + |P_y| m\Omega\delta}{m\Omega\delta p_-} \right). \quad (7)$$

We take the opportunity to thank L. P. Gor'kov, Yu. N. Ovchinnikov, and G. M. Eliashberg for a useful discussion of the work.

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