CAN ELECTROMAGNETIC INTERACTIONS OF W BOSONS CUT OFF WEAK INTERACTIONS?

B. L. Ioffe
Submitted 26 May 1969
ZhETF Pis. Red. 10, No. 1, 53 - 55 (5 July 1969)

Estimates of effects of higher order in weak interactions show [1] that the increase of the weak-process amplitudes with increasing energy, which appears in the usual theory of weak interaction, cannot occur up to energies $\Lambda_W \sim G^{-1/2}$ such that the weak interaction becomes strong, and should stop at much lower energies. In hadron-lepton processes, an exact account of the strong interactions does not lead to cutoff of the diverging integrals. In [3] we advanced the hypothesis that the divergences of the weak amplitudes may be cut off in the theory with an intermediate boson at energies $\Lambda_e \sim \mu/\varepsilon << \Lambda_W$ (μ - mass of W, $e^2 = 1/137$), as a result of the W-boson electromagnetic interactions, which become strong at these energies. In the present paper we attempt to ascertain, on the basis of an analysis of the perturbation-theory diagrams, whether allowance for the electromagnetic interactions of the W bosons can lead to a cutoff of the divergences in the amplitudes of the weak processes.

We consider first the correction of order g^2 (g - W-boson weak-interaction constant, $4\pi g^2/\mu^2 = G/\sqrt{2}$), which determines the renormalization of the lepton wave function with allowance for the electromagnetic interactions of the leptons and W bosons (without an anomalous magnetic moment). The corresponding Feynman diagram is shown in Fig. 1, where $G_{\mu\nu}(k)$ is the exact Green's function of the W boson and $\Gamma_\mu(p, p - k; k)$ is the exact vertex part for the emission of the W. (It is obvious that the lepton Green's function can be regarded as free, inasmuch as the radiative corrections to it are of the order of $e^2 \ln k^2/m^2)^n$, i.e., inessential when $k \sim \Lambda_e$.) By virtue of the Kallen-Lehmann representation describing $G_{\mu\nu}(k)$, the greatest degree of divergence should be expected when account is taken of the longitudinal part of $G_{\mu\nu}(k)$, i.e., of the terms proportional to $k_\mu k_\nu$. It follows therefore that the cutoff of the devergences can occur in the given diagram only in the case when, at large values of k, the longitudinal part of $\Gamma_\mu(p, p - k; k)$ in the n-th approximation is of the order of $(e^2\bar{k}^2/\mu^2)^m(e^2\Lambda_e^2/\mu^2)^p$, $m + p = n > 0$ (if we assume the cutoff momentum to be $\Lambda_e \sim \mu/e$, i.e., $e^2/\Lambda_e^2/\mu^2 \sim 1$). Actually, however, the longitudinal part of $\Gamma_\mu(p, p - k; k)$ in the n-th approximation turns out to be of the order of

$$\Gamma^n_{\mu\,long} \sim e^2 \left(\frac{e^2 k^2}{\mu^2}\right)^m \left(\frac{e^2 \Lambda_e^2}{\mu^2}\right)^p \ln \frac{\Lambda^2}{\mu^2}, \quad m + p + 1 = n \tag{1}$$

i.e., the radiative corrections of the longitudinal part of Γ_μ are small when $k^2 \sim \Lambda_e^2$ and consequently cannot lead to a cutoff of the devergence.

To prove formula (1), let us consider first the e^2-approximation correction to Γ_μ, described by the diagram of Fig. 2 and equal to

$$\Gamma_\mu^{(1)} = \frac{e^2}{4\pi^3 i} \int d^4k' \gamma_\Lambda (p - k + k' - m)^{-1} \gamma_\sigma (1 + \gamma_5) \frac{1}{k'^2} \times$$
$$\times [(k - k')^2 - \mu^2]^{-1} [\delta_{\sigma\nu} - (k - k')_\sigma (k - k')_\nu/\mu^2] \gamma^\lambda_{\nu\mu} (k - k', k), \tag{2}$$

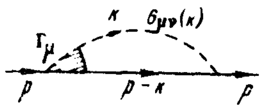


Fig. 1

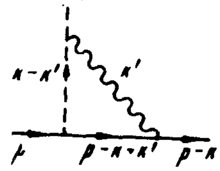


Fig. 2

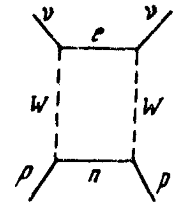


Fig. 3

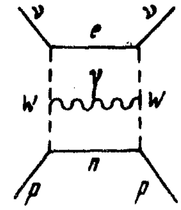


Fig. 4

where

$$\gamma^{\lambda}_{\nu\mu}(k_2,k_1) = (k_{1\lambda}+k_{2\lambda})\,\delta_{\nu\mu} - k_{1\nu}\delta_{\mu\lambda} - k_{2\mu}\,\delta_{\nu\lambda} \qquad (3)$$

is the vertex part for the interaction of W with the electromagnetic field. By virtue of the condition

$$k_{2\nu}\,\gamma^{\lambda}_{\nu\mu}(k_2 k_1)\,k_{1\mu} = 0 \qquad (4)$$

which follows from (3), the term $(k - k')_\sigma(k - k')_\mu/\mu^2$ in the W-boson Green's function in (2) makes no contribution to the expression for the longitudinal part of $\Gamma^{(1)}_\mu$, so that $\Gamma^{(1)}_{long}$ is of the order of $e^2\ln(\lambda^2/k^2)$. Using relation (4) and the method given in (3), we can generalize the proof to the case of the n-th approximation; this leads to the estimate (1).

We now consider the question of whether W-boson electromagnetic interactions can cut off the divergences in a diagram such as in Fig. 3, which leads to the occurrence of neutral currents in the hadron-lepton interaction (we disregard strong interactions). Since, as we have shown, the change of the vertices due to the electromagnetic interaction does not lead to cutoff in the first approximation in e^2, only the diagram of Fig. 4 can make a contribution. The estimate can be obtained in elementary fashion and yields

$$\frac{\text{Contribution of diagram of Fig. 4}}{\text{Contribution of diagram of Fig. 3}} \sim e^2. \qquad (5)$$

if it is assumed that all the integration momenta are bounded by the condition $k^2 < \Lambda_e^2$. In the presence cutoff due to the electromagnetic interaction, it would be natural to assume the contribution of the radiative corrections to be of the same order as the contribution of the main term, so that the estimate (5) also argues against cutoff due to electromagnetic interactions of W bosons.

The foregoing reasoning, strictly speaking, does not apply to hadron-lepton processes, since it does not take strong interactions into account. It is difficult to imagine, however, that strong interaction can greatly increase (by a factor e^{-2}) the contribution of the radiative corrections and cut off the integrals at momenta of the order of Λ_e.

Although all the foregoing arguments contain a number of assumptions, little hope is left that electromagnetic interactions of W bosons can cut off the growth of the weak interactions.

The fact that the W boson has an anomalous magnetic moment κ does not change this conclusion, for in this case the cutoff should occur at an energy $\Lambda_e \sim \mu\,(\kappa e^2)^{-1/4}$, and the radiative corrections turn out to be of the order of $\kappa e^2 \Lambda_e^2/\mu^2 \sim (\kappa e^2)^{1/2}$, i.e., they are again small.

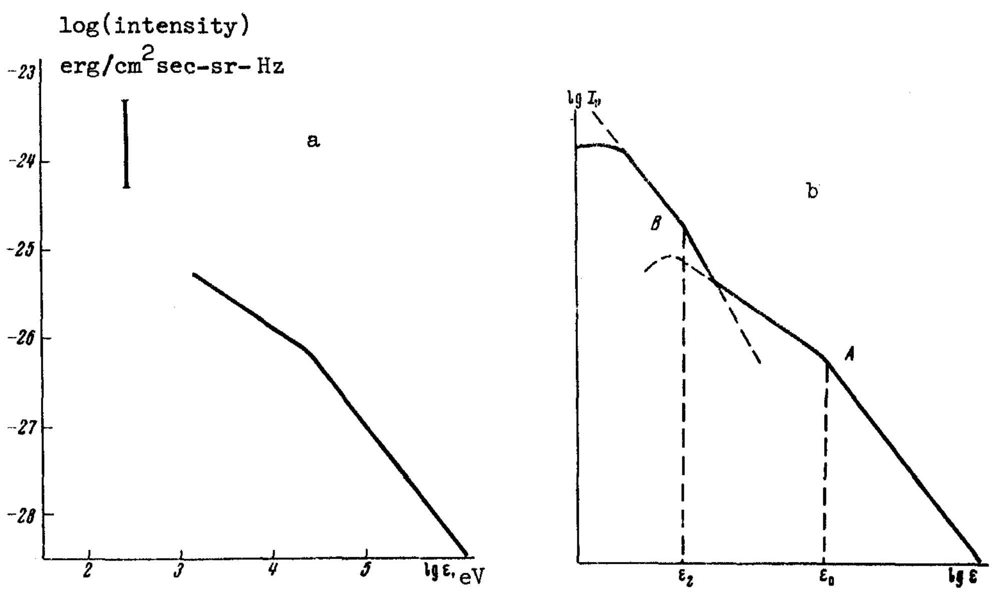
I am grateful to B. V. Geshkenbein for useful discussions.

[1] B. L. Ioffe and E. P. Shabalin, Yad. Fiz. 6, 828 (1967) [Sov. J. Nuc. Phys. 6, 603 (1968)]; ZhETF Pis. Red. 6, 978 (1967) [JETP Lett. 6, 390 (1967)].
[2] M. B. Halpern and G. Segre, Phys. Rev. Lett. 19, 611 1000(E) (1967).
[3] B. L. Ioffe, Zh. Eksp. Teor. Fiz. 47, 975 (1964) [Sov. Phys.-JETP 20, 654 (1965)].

ORIGIN OF THE BACKGROUND X-RADIATION

M. S. Longair [1)] and R. A. Syunyaev [2)]
Institute of Applied Mathematics, USSR Academy of Sciences
Submitted 4 June 1969
ZhETF Pis. Red. 10, No. 1, 56 - 59 (5 July 1969)

Observation of the isotropic background x-radiation has revealed a number of distinctive singularities in its spectrum [1, 2]. This spectrum (Fig. a) cannot be described by a single power-law formula $I(\varepsilon) \sim \varepsilon^{-\alpha}$, but for photons with energy $\varepsilon \geq 1$ keV there are two laws, viz., $\alpha \simeq 0.7$ at low energies and $\alpha \simeq 1.2$ at high ones, the kink occurring in the region $\varepsilon \sim 20$ - 40 keV. Below 1 keV, the exact flux is unknown, since we do not know how strongly it is absorbed by the interstellar gas in our galaxy. Even without allowance for the absorption, however, observations have shown that the soft x-radiation with $\varepsilon \sim 280$ eV exceeds the value expected by extrapolation from the energy region $1.5 < \varepsilon < 20$ keV. If the spectrum obeys a power law in this region, then its slope should exceed $\alpha \gtrsim 1.2$.



Spectrum of background x-radiation: a) observed, b) expected in accord with the model described in this paper.

---
1) Cambridge University and Physics Institute, USSR Academy of Sciences
2) Institute of Applied Mathematics.