conducting state and the transition in the magnetic field become greatly diffused at this pressure, possibly as a result of the fact that the sample is not single-phase. This phase has apparently a positive value of dT_c/dP. At pressures below 100 kbar, no superconductivity is observed above 0.1°K.

[1] N. B. Brandt and N. I. Ginzburg, Fiz. Tverd. Tela 3, 3461 (1961) [Sov. Phys.-Solid State 3, 2510 (1962)].
[2] T. R. R. McDonald, E. Gregory, C. S. Barberich, D. B. McWhan, T. H. Geballe, and G. W. Hull, Phys. Rev. Lett. 14, 16 (1965).
[3] I. V. Berman and N. B. Brandt, ZhETF Pis. Red. 7, 412 (1968) [JETP Lett. 7, 323 (1968)].
[4] J. Wittig and B. T. Matthias, Science, 160, 994 (1968).
[5] P. W. Bridgman, Proc. Amer. Acad. Arts. Sci. 81, 165 (1932).
[6] J. Wittig, Solid. State Comm. 7, No. 5 (1969).

DIVERGENCES OF AMPLITUDES OF WEAK NONLEPTONIC PROCESSES IN THE THEORY WITH AN INTERMEDIATE BOSON

A. I. Vainshtein and B. L. Ioffe
Submitted 27 March 1969; resubmitted 2 June 1969
ZhETF Pis. Red. 10, No. 2, 91 - 95 (20 July 1969)

The use of current algebra in the analysis of the amplitudes of weak hadron-lepton processes in the higher orders in the weak interaction has shown [1] that in some cases an exact allowance for the strong interactions does not lead to an effective cutoff of the integrals, which remain divergent and can be cut off, by the same token, only via weak or electromagnetic interactions.

In the case of nonleptonic processes, in the theory with an intermediate boson, the situation is less clear, but here, too, there are grounds for assuming [2 - 4] that the strong interactions do not cut off the integrals with respect to the momenta of the virtual W bosons. To eliminate the resultant difficulties, it was proposed in [3 - 5] that the Hamiltonian of the interaction is the sum of the following terms: symmetrical in SU(3) × SU(3) and SU(3) × SU(3) symmetry breaking terms that transform in accordance with the representations $(3, \bar{3})$ and $(\bar{3}, 3)$ of the SU(3) × SU(3) groups. Then, as shown in [3], the contribution of the diverging terms to the amplitudes of the transitions with $\Delta S = 1$ (and also to the parity-nonconversing amplitudes with $\Delta S = 0$) vanish rigorously.

In this paper we wish to present an estimate of the diverging terms in the amplitudes of weak nonleptonic processes, without using the assumption made in [3 - 5], and to find the resultant values of the cutoff parameter. The main assumption used here will be the hypothesis of partial conservation of the partial current without change of strangeness (PCAC) or more accurately, the assumption that in the limit of zero pion mass the axial current with $\Delta S = 0$ is rigorously conserved.

The matrix element of the transition between the hadron states a and b due to emission and absorption of an intermediate W boson can be written in the form

$$M(2\pi)^4\delta^4(p_a - p_b) = \frac{4\pi g^2}{(2\pi)^4}\int d^4k\,\frac{1}{k^2\mu^2}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{\mu^2}\right) \times \tag{1}$$
$$\times \int d^4x\,d^4y\,e^{ik(x-y)} < b\,|\,T\,\{j_\mu^+(x),\;\; j_\nu(y)\}\,|\,a>,$$

where $4\pi g^2/\mu^2 = G\sqrt{2}$ and j_μ is the weak hadron current. The quadratically divergent term in in (1) is of the form[1)]

$$M_{\text{div}}(2\pi)^4\,\delta^4(p_a - p_b) = \frac{g^2}{4\pi^3\mu^2}\int\frac{d^4k}{k^2\mu^2}\times \tag{2}$$
$$\times\int d^4x\,d^4y\,e^{ik(x-y)}\,\delta(x_o - y_o) < b\,|[\partial_\mu j_\mu^+(x),\ j_o(y)]|\,a >.$$

We shall assume that the matrix element of the equal-time commutator $<b\,|[\partial_\mu j_\mu(x),\ j_o(y)]\,|a>$ contains only terms proportional to $\delta'(\vec{x} - \vec{y})$ and $\delta'(\vec{x} - \vec{y})$ [sic!]. Since the terms proportional to $\delta'(\vec{x} - \vec{y})$ make no contribution to (2), we can write

$$M_{\text{div}}(2\pi)^4\delta(p_a - p_b) = \frac{q^2}{4\pi^3\mu^2}\int\frac{d^4k}{k^2-\mu^2}\int d^4x\,d^4y\,\delta(x_o - y_o)\ \times$$
$$\times <b\,|[\partial_\mu j_\mu^+(x),\ j_o(y)]|\,a> = -i\,\frac{g^2\Lambda^2}{4\pi\mu^2}\int d^4x <b\,|[\partial_\mu\, j_\mu^+(x),\, Q(x_o)]|a>, \tag{3}$$

where Λ is the cutoff limit and $Q(x_o) = \int dx\; j_o(x,\ x_o)$.

Let us consider the most interesting transition with $|\Delta S| = 1$, and assume for concreteness that $\Delta S = S_b - S_a = 1$. Then the only terms making a nonvanishing contribution to (3) are those with $\Delta S = 0$ from $\partial_\mu j_\mu^+(x)$ and those with $\Delta S = 1$ from $Q(x_o)$. The main contribution to the divergence of the weak current without change of strangeness is made by the axial current. If it is assumed that the axial current without change of strangeness is conserved rigorously in the limit of zero pion mass, then $\partial_\mu j_\mu^+$ will be of the order of $\partial_\mu j_\mu^+ = \partial_\mu A_\mu^+ \sim (\mu_\pi/m_o)^2$, where m_o is a certain effective mass[2)]. We thus get for the diverging part of the matrix element of the transition with $\Delta S = 1$ the estimate[3)]

$$M_{\text{div}} \sim \frac{g^2\Lambda^2}{4\pi\mu^2}\sin\theta\,\left(\frac{\mu\pi}{m_o}\right)^2 = \frac{G}{(4\pi)^2\sqrt{2}}\sin\theta\,\Lambda^2\left(\frac{\mu\pi}{m_o}\right)^2, \tag{4}$$

where θ is the Cabibbo angle.

The effective parameter of the partial conservation of the axial current will be assumed

---

[1)] As usual [3 - 5], it is assumed that $\int d^4x\, e^{ikx} < b|T\{\partial_\mu j_\mu(x)\partial_\nu j_\nu(0)\}|a>$ decreases when $k \to \infty$, and 2) the Schwinger terms are c-numbers. The latter assumption is very important, for if it is not satisfied nonvanishing terms that are proportional to $g^2\Lambda^4$. It should be noted that assumption 2), in particular, denotes that either there is no direct interaction between the W and the field ϕ^+ of the π and K mesons, or else that this interaction contains term of the form $W^+W^-\phi^+\phi$, which cancel completely the contribution of the Schwinger terms.

[2)] Such an estimate holds true in all cases, with the exception of matrix elements containing pole diagrams with single-pion intermediate states. It can be shown that these diagrams make no contribution to (3).

[3)] The diverging part of the matrix element of the $K \to 2\pi$ decay was calculated in [6], but the calculated result was not proportional to the small parameter $(\mu_\pi/m_0)^2$. The difference between the results of [6] and of our work lies in the fact that the author of [6] applied the approximate method of the PCAC hypothesis to a zero matrix element $<2\pi\partial_\mu j_\mu|K>$ and obtained for it a nonzero result.

equal to $(\mu_\pi/m_o)^2 \sim 0.1$, i.e., $m_o = 0.5$ GeV. We then get for M_{div}

$$M_{div} \sim 5 \cdot 10^{-4}\, G\,\Lambda^2 \sin\theta. \tag{5}$$

Comparing M_{div} with the contribution of the nondiverging terms, which should be of order $M_{conv} \sim Gm_o \sin\theta$ for the cutoff limit Λ, we get the limitation

$$\Lambda \leq 25 \text{ GeV}. \tag{6}$$

We now consider transitions with $\Delta S = 0$ and with parity change $P = -1$. In this case one of the weak currents in (1) should be the vector, and the other axial, and we can write the quadratically diverging part of (1) in the form (3) in such a way as to include a divergence of the vector current $\partial_\mu V_\mu^+$.[4] If neither current changes strangeness, then the order of smallness of $\partial_\mu V_\mu^+$ is determined by the violation of the isotopic invariance, i.e., by the electromagnetic interactions, and we get for the ratio M_{div}/M_{conv} the estimate

$$\frac{M_{div}}{M_{conv}} \sim \frac{g^2\Lambda^2}{4\pi\mu^2}\,\frac{e^2}{\pi}/\,G m_o^2 = \frac{e^2}{\sqrt{2}(4\pi)^2\pi}\left(\frac{\Lambda}{m_o}\right)^2 \leqslant 1,\quad \Lambda \lesssim 150 \text{ GeV}. \tag{7}$$

On the other hand, if both currents change strangeness, than the order of smallness of $\partial_\mu V_\mu^+$ is determined by the breaking of the SU(3) symmetry, and we get

$$\frac{M_{div}}{M_{conv}} \sim \frac{g^2\Lambda^2}{4\pi\mu^2}\sin\theta\,\lambda/G m_o^2 \sim 2\cdot 10^{-4}\left(\frac{\Lambda}{m_o}\right)^2\lambda \leqslant 1,\quad \Lambda \lesssim 150 \text{ GeV}, \tag{8}$$

where λ is a small parameter characterizing the violation of the SU(3) invariance, $\lambda \simeq 0.2$.

Let us estimate, finally, the matrix elements of the transitions with $\Delta S = 0$, $\Delta T = 1$, $P = +1$. Such transitions may be due to allowance for the electromagnetic interaction, so that the matrix element (3) should be equated with $M_{conv} \sim e^2/\pi$, so that

$$\frac{M_{div}}{M_{conv}} \sim \frac{G\Lambda^2}{(4\pi)^2}\,\frac{\pi}{e^2}\left(\frac{\mu_\pi}{m_o}\right)^2 \sim 2\cdot 10^{-6}\left(\frac{\Lambda}{m}\right)^2 \lesssim 1,\quad \Lambda \lesssim 600 \text{ GeV}. \tag{9}$$

Transitions with $\Delta S = 0$, $\Delta T = 0$, and $P = +1$ are of no interest, since they are indistinguishable from transitions due to strong interactions.

The obtained estimates of the cutoff limit Λ, (6) - (8), are close to those following from an examination of leptonic decays of hadrons [1].

We are grateful to V. I. Zakharov, V. I. Ogievetskii, V. V. Sokolov, and I. B. Khriplovich for useful discussions.

[1] B. L. Ioffe and E. P. Shabalin, Yad. Fiz. 6, 828 (1967) [Sov. J. Nuc. Phys. 6, 603 (1968)]; ZhETF Pis. Red. 6, 978 (1967) [JETP Lett. 6, 390 (1967)].
[2] M. B. Halpern and G. Segre, Phys. Rev. Lett. 19, 611 1000(E) (1967).
[3] C. Bouchiat, J. Iliopouicz, and J. Prentki, Nuovo Cim. 56A, 1150 (1968).
[4] R. Gatto, G. Sartori, and M. Tonin, Phys. Lett. 28B, 129 (1968).
[5] N. Cabibbo and L. Maiani, Phys. Lett. 28B, 131 (1968).
[6] G. Venturi, Phys. Rev. 174, 2154 (1968).

[4] Strictly speaking, for transitions with $\Delta S = 0$ formulas (2) and (3) should be asymmetrized with respect to the currents j^+ and j.