"BANANA" SELF FOCUSING OF BEAMS

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Most papers of self-focusing (see [1 - 4], the review [5] and the literature therein) dealt with cases when the refractive index n was increased by the field of the beam ($\delta n > 0$), and the beam intensity decreased from the axis towards the edge ($\partial I/\partial r < 0$), thus ensuring the necessary self-focusing condition $\partial n/\partial r < 0$ for the entire cross section of the beam.

However, in many widely used media of practical interest, heating leads to $\delta n < 0$ (for example in gases and liquids such as air and water), so that usually $dn/dT < 0$ (if the pressure has time to become equalized), or else $\partial n/\partial T|_\rho < 0$ (for short instants of time $t \leq t_s = a/c_s$), so that ordinary beams having an intensity that decreases away from the axis should become defocused (since $\partial n/\partial r > 0$).

We report here for the first time the achievement of self focusing in water, and show that for a large class of beams with a nonmonotonic intensity distribution (a rise from the axis towards the periphery and a sharp decrease beyond a certain radius r_m) self-focusing of the part of the beam with $r < r_m$ should be observed, and a loss of the lateral part of the beam with $r > r_m$ (the beam is peeled off like a banana). The lost part of the beam energy can be neglected in the case of a steep fall-off (when $\int_{r_m}^{\infty} Ir\,dr \ll \int_0^{r_m} Ir\,dr$), especially in the presence of absorption or scattering of the radiation in the medium (we are interested just in dissipative media and in dissipative self-focusing).

1. Let us obtain the refraction profile, assuming that the reaction of the refraction on the beam distribution is weak (such an assumption is usually made [6] in the estimate of the initial stage of self focusing).

Assume that the derivative of the refractive index of the medium with respect to the temperature is given, $n'_T < 0$. We then obtain for the change of the refractive index in the case of a beam with an intensity distribution $I(r, t)$

$$\delta n(r,t) = \frac{n'_T \kappa}{C\rho} \int_0^t I(r,t)\,dt = \frac{n'_T \kappa}{C\rho}\, q(r,t) = -\alpha q(r,t),$$

where C is the specific heat of the medium, ρ its density, κ the absorption coefficient of the light, and $q(r, t)$ the density of the transmitted light energy. From the self-focusing condition $(\partial/\partial r)\delta n < 0$ we obtain $\alpha(\partial q/\partial r) > 0$, i.e., the intensity of the light should increase radially.

For a specified profile $\delta n(r)$, the change of the angle of the inclination of the rays is $(\partial\theta/\partial z) \approx (\partial n/\partial r) = \partial\delta n/\partial r$; let us consider particular focusing cases and the corresponding intensity distributions.

In order for a parallel beam incident on a layer of medium of thickness L to be focused over a distance L_f(>> L) it is necessary to have

$$\delta\theta \simeq (r/L_f) \simeq Ln'_r, \text{ i.e. } L_f \simeq r/Ln'_r.$$

Since this condition is satisfied for all r, we get $n'_r = A(t)r$, or

$$\delta n = \frac{1}{2} A(t)r^2 + B(t) = n'_T \delta T = \frac{n'_T \kappa}{C\rho} \int_o^t I dt,$$

i.e., $B(t) = a\int_o^t I(o,t)\,dt.$ Since usually

$I(r,t) = I_m \phi(r)\psi(t)$ we get $\frac{1}{2} A(t)r^2 =$

$$= aI_m \{\phi(r) - \phi(o)\} \int_o^t \psi(t)dt,$$

i.e., $\phi(r) = \phi(o) + (r/a)^2$ and $A(t) = \frac{2aI_m}{a^2} \int_o^t \psi(t)\,dt,$

where a is the effective radius of the beam, and ϕ and ψ are dimensionless functions. Consequently

$$L_f(t) = \frac{1}{LA(t)} = \frac{a^2}{2aLI_m \int_o^t \psi(t)dt}$$

It is customary to put $\psi(t) = 1$ for $0 < T$ (rectangular pulse), i.e., $\int_0^t \psi(t)dt = t$, or $\psi(t) = t/T_m$ for $0 < t < T_m$ (in this case $\int_0^t \psi(t)dt = t^2/2T_m$); or else $\psi(t) = \sin(\pi t/2T)$; $t < 2T_m$, and then

$$\int_o^t \psi(t)dt = \frac{2T_m}{\pi}\left(1 - \cos\frac{\pi t}{2T_m}\right), \text{ i.e., in these cases } L_f \sim 1/t;$$

$$\sim \frac{1}{t^2}; \qquad \sim \frac{1}{\sin^2(\pi t/4T_m)},$$

i.e., the focal point draws a trace from infinity to the layer of the medium.

2. An experiment was set up to confirm the process under consideration. The experimental setup is shown in Fig. 1. A ruby laser was used, without Q-switching, with energy 25 J in a millisecond pulse. The required beam profile was formed by passing the beam through a glass plate with a screen at the center of the beam (the screen was a sphere or a disc of 1.3 mm diameter). A liquid-filled cell, 15 cm long, was placed 70 cm away from the screen, and the image of the beam, after passing through several light filters, was recorded on photographic film at a distance of 70 cm from the cell. (At this distance from the cell, the shadow of the screen was smoothed out, and a smooth distribution with a minimum at the center

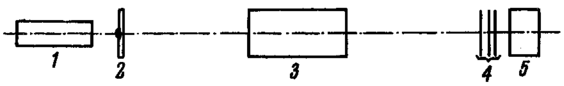

Fig. 1. 1 - Laser, 2 - glass plate with screen, 3 - cell with water, 4 - set of filters, 5 - camera of high-speed image camera (SFR).

was obtained). When an absorbing medium (a solution of cuprous oxide) was added to the liquid in the cell, the image of the beam changed abruptly - a spot of increased intensity appeared in the center. Figure 2 shows the images obtained for the case when the cell contained pure water (a) and water colored with cuprous oxide (b and c, the absorption length was approximately equal to the length of the cell; photo c was obtained with a high-speed streak camera).

The considered self-focusing processes[1)] can extend the class of media and the range of conditions for which self-focusing of optical, radio, and ultrasonic beams becomes possible in either the pulsed or stationary regime. A special choice of the beam profile of powerful gas or solid-state lasers can greatly influence their divergence in water and air. This phenomenon can arise spontaneously in the presence of dips in the intensity distribution, or after passing through absorbing centers in the medium.

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[7] G. A. Askar'yan, ibid. 54, 1393 (1968) [Sic!]

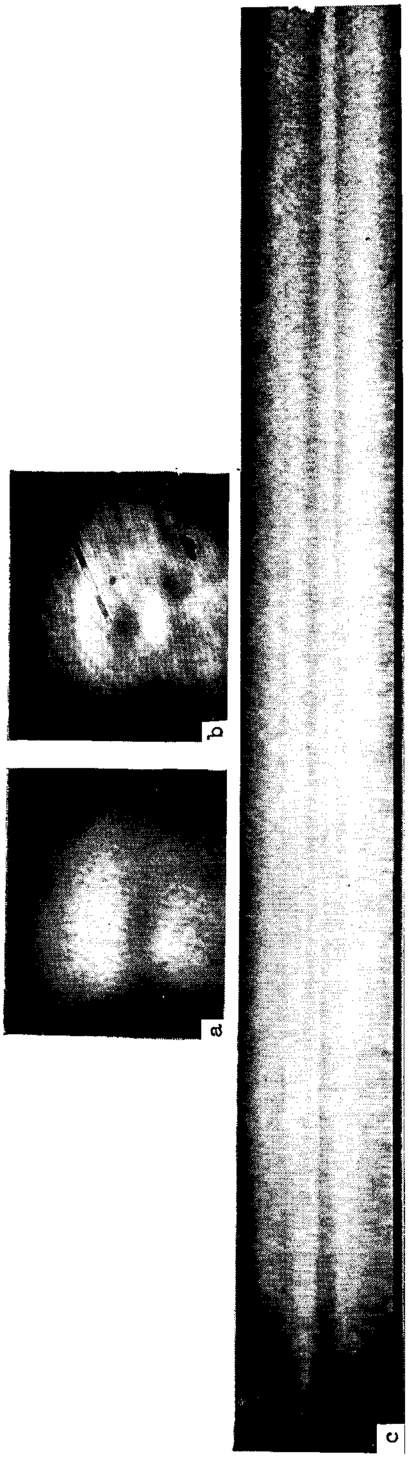

Fig. 2. Beam photograph: a - water without absorbing additive, b - water with slight amount of cuprous oxide, c - the same on moving film of SFR camera (the shown part of the image corresponds to a time of 600 μsec.

[1)] The profile n(r), which remains after the passage of the beam, can be used to decrease the divergence of another beam with smaller radius (in this case waveguide transmission or focusing can be obtained without lateral losses; see [7] concerning tubular optical waveguide beams).