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LONGITUDINAL DISTANCES IN GAMMA QUANTUM SCATTERING AND ASYMPTOTIC ELECTROPRODUCTION CROSS SECTIONS

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1. It was shown in an earlier paper [1] that the question of the longitudinal distances that play a role in the scattering of virtual gamma quanta by nucleons at high energies can be clarified by analyzing the dependence of the imaginary part of the amplitude for the forward scattering of a virtual gamma quantum with mass q^2 by a nucleon, $\mathrm{Im}M_{\mu\nu}(\nu, q^2)$ on q^2 ($\nu = pq$, p and q are the momenta of the nucleon and of the gamma quantum). Namely, if $\mathrm{Im}M_{\mu\nu}(\nu, q^2)$ changes appreciably with changing q^2 at $\nu \gg m^2$ and $\nu \gg |q^2|$ and at a fixed ν, then large longitudinal distances, which increase linearly with increasing energy, play a role in the scattering of the gamma quanta by nucleons at high energy. But if $\mathrm{Im}M_{\mu\nu}(\nu, q^2)$ is independent of q^2 under the same conditions, then the principal role is played by finite longitudinal distances (or distances increasing more slowly than ν). When $q^2 < 0$, the value of $\mathrm{Im}M_{\mu\nu}(\nu, q^2$ averaged over the spins of the nucleon is expressed in the following manner in terms of the invariant functions $w_1(\nu, q^2)$ and $w_2(\nu, q^2)$, which determine the total cross section for the electroproduction of hadrons on nucleons:

$$\mathrm{Im}\, M_{\mu\nu}(\nu, q^2) = \frac{1}{m^2}\left(p_\mu - \frac{\nu q_\mu}{q^2}\right)\left(p_\nu - \frac{\nu q_\nu}{q^2}\right) w_2(\nu, q^2) - \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) w_1(\nu, q^2). \tag{1}$$

Hence

$$w_1 = \frac{1}{2}\,\mathrm{Im}\, M_T\,; \qquad w_2 = -\frac{q^2 m^2}{\nu^2 - m^2 q^2}\left(\frac{1}{2}\,\mathrm{Im}\, M_T - \frac{m^2 q^2}{\nu^2}\,\mathrm{Im}\, M_L\right) \tag{2}$$

where $M_T = M_{xx} + M_{yy}$, $M_L = M_{zz}$, and the z axis is directed along the gamma-quantum momentum. Experimental data on electroproduction on protons [2] show that in a certain interval of variation of q^2, at fixed ν satisfying the conditions $\nu \gg m^2$ and $\nu \gg |q^2|$, the function $w_2(\nu, q^2)$ does not depend on q^2, namely $w_2 \approx 0.3\ m/\nu$. By virtue of (2), this means that $\mathrm{Im}M_T$ and (or) $\mathrm{Im}M_L$ vary appreciably with varying q^2, i.e., large longitudinal distances play a role in the scattering of gamma quanta by nucleons.[1)]

Another argument in favor of the role of large longitudinal distances arises when the data on the energy dependence of the total cross sections for photoproduction on protons and for the absorption of neutrinos and antineutrinos by nucleons are compared. The total cross

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1) In the author's paper [1], the behavior of $w_2(\nu, q^2)$ as a function of ν and q^2 was erroneously identified with the behavior of $\mathrm{Im}M_{\mu\nu}(\nu, q^2)$. As a result, in the absence of a dependence of w_2 on q^2, it was incorrectly concluded that the final states play the main role in the scattering of gamma quanta by nucleons.

sections for photoproduction on protons and for the absorption of neutrinos and antineutrinos by nucleons are compared. The total cross section for the production of hadrons by protons is given by

$$\sigma_\gamma = 4\pi^2 e^2 (m/\nu)\, w_1^{ep} (\nu,\ 0). \tag{3}$$

At high energies, experiment [3] yields $\sigma_\gamma \sim$ const, so that $w_1^{ep}(\nu,\ 0) \approx \nu/m^3$. It is natural to assume that the cross sections for the production by isovector and isoscalar photons should be of the same order, and consequently $w_1^{ep,V}(\nu,\ 0) \sim \nu/m^3$ for isovector photons, too. The total cross section for the absorption of a neutrino (antineutrino) by a nucleon can be written in the form (cf. e.g., [4])

$$\frac{d\sigma^{\nu(\bar\nu)}}{dq^2 d\nu} = \frac{E'}{E}\frac{G^2}{2\pi m}\Big[\cos^2\frac{\theta}{2} w_2^{\nu(\bar\nu)}(\nu,\ q^2) + 2\sin^2\frac{\theta}{2} w_1^{\nu(\bar\nu)}(\nu,\ q^2) \pm \frac{E+E'}{m}\sin^2\frac{\theta}{2} w_3^{\nu(\bar\nu)}(\nu,\ q^2)\Big], \tag{4}$$

where E is the neutrino (antineutrino) energy, and E' and θ are the energy and emission angle of the muon. By virtue of the isotopic invariance, we have for the contribution of the vector current

$$(w_i^{\nu p} + w_i^{\nu n} + w_i^{\bar\nu p} + w_i^{\bar\nu n})/4 = 2 w_i^{epV}.$$

We assume now that the finite longitudinal distances play a role in the imaginary part of the scattering due to the vector current $\mathrm{Im}M_{\mu\nu}$. When they have for $|q^2|/\nu \ll 1$

$$w_1^{ep,V}(\nu,\ q^2) \approx w_1^{ep,V}(\nu,\ 0) \sim \nu/m^3.$$

Allowing in (4) only for the contribution $w_1(\nu,\ q^2)$ due to the vector integration and integrating with respect to $|q^2|$ in the range $0 < |q^2|/2\nu < \gamma \ll 1$, we obtain the estimate

$$(\sigma_{\nu p} + \sigma_{\nu n} + \sigma_{\bar\nu p} + \sigma_{\bar\nu n})/4 \gtrsim (G^3/4\pi)\gamma^2 E^2. \tag{5}$$

The experimental data on the total cross section for the scattering of neutrinos by nuclei, obtained with accelerators [5] and with cosmic rays [6], do not agree with (5) and by the same token exclude the assumption that finite distances play an appreciable role.

2. Let us consider the asymptotic behavior of $\mathrm{Im}M_{\mu\nu}(\nu,\ q^2)$ when $\nu \to \infty$, $|q^2| \to \infty$, and $\omega = |q^2|/\nu =$ const. We can represent $\mathrm{Im}M_{T,L}(\nu,\ q^2)$ in the form

$$\mathrm{Im}\,M_{\mu\nu}(\nu,\ q^2) = \frac{1}{4}\int d^4x\, e^{iqx} f_{\mu\nu}(x) = \frac{i\pi}{2}\int_0^\infty d\tau \int_0^\infty ds \int_0^{2s} d\rho^2 \sin\Big(\frac{\nu}{m}\tau - \frac{m\omega s}{2}\Big) f_{\mu\nu}(x), \tag{6}$$

$$f_{\mu\nu}(x) = \sum_\lambda \langle p,\ \lambda |\ [j_\mu(x),\ j_\nu(0)]\ | p,\ \lambda\rangle,$$

where λ is the nucleon spin projection, $\tau = t - z$, $s = (t + z)/2$, and ρ is the transverse distance. The general expression for $f_T(x)$ and $f_L(x)$ in the laboratory frame is

$$f_T(x) = A(x^2, t) + B(x^2, t)\rho^2, \quad f_L(x) = A(x^2, t) + B(x^2, t) z^2, \tag{7}$$

where $A(x^2, t)$ and $B(x^2, t)$ are functions of x^2 and t only. When $\nu \to \infty$ and ω = const, the behavior of $\mathrm{Im} M_{T,L}(\nu, \omega)$ is determined by the behavior of $f_{T,L}(x)$ at small values of $x^2 \sim 1/\nu\omega \to 0$ (here $t \approx 1/m\omega$). If we assume the simplest assumption concerning the behavior of $A(x^2, t)$ and $B(x^2, t)$ as $x^2 \to 0$, namely

$$A(x^2, t) = (x^2 m^2)^{-\gamma_A} \phi_A(t), \quad B(x^2, t) = (x^2 m^2)^{-\gamma_B} \phi_B(t),$$

then we get from (6)

$$\mathrm{Im} M_T(\nu, q^2) = \left(\frac{m^2}{\nu}\right)^{\gamma_T} F_T(\omega), \quad \mathrm{Im} M_L(\nu, q^2) = \left(\frac{m^2}{\nu}\right)^{\gamma_L} F_L(\omega), \tag{8}$$

where

$$2 - \gamma_T = \max(\gamma_A, \gamma_B - 2), \quad 2 - \gamma_L = \max(\gamma_A, \gamma_B).$$

Thus, the functions $w_1(\nu, q^2)$ and $w_2(\nu, q^2)$ take the form

$$w_1(\nu, q^2) = \frac{1}{2}\left(\frac{m^2}{\nu}\right)^{\gamma_T} F_T(\omega), \quad w_2(\nu, q^2) = \frac{\omega}{\nu}\left[\frac{1}{2}\left(\frac{m^2}{\nu}\right)^{\gamma_T} F_T(\omega) + \left(\frac{m^2}{\nu}\right)^{\gamma_L + 1} \omega F_L(\omega)\right]. \tag{9}$$

From the condition that the integral with respect to ρ^2 in (6) be convergent, it follows that $\gamma_A < 1$ and $\gamma_B < 1$, i.e, $\gamma_T > 1$ and $\gamma_L > 1$. If the behavior of A and B as $x^2 \to 0$ is of the form $(x^2)^{\gamma(t)}\phi(t)$, then the asymptotic forms of w_1 and w_2 differ from those in (9) by a factor proportional to a certain power of $\ln(\nu/m^2)$. A slower decrease of w_1 and w_2 as functions of ν can be obtained by assuming that A and B are proportional to $\delta(x^2)$ or to its derivatives when $x^2 \to 0$. Thus, for example, when $A(x^2, t) = \phi_A(t)\partial\delta(x^2)/\partial t$ or $Bz^2 = \phi_B(t) \times \partial^2\delta(x^2)/\partial z^2$, we get for w_1 and w_2 the expressions in (9) with $\gamma_T = 0$ and $\gamma_L = -1$, respectively. It is not clear, however, how it is possible to obtain non-integer γ_T and γ_L in the intervals $0 < \gamma_T < 1$ and $-1 < \gamma_L < 1$.

Definite values of γ_T and γ_L are obtained if it is assumed that Adler's sum rule holds [7, 8]

$$\int_{|q^2|/2}^{\infty} d\nu\, w_2^a(\nu, q^2) = 1, \quad w_2^a = w_{2,V}^{\bar{\nu}p} - w_{2,V}^{\nu p} = 2(2w_2^{1/2} - w_2^{(3/2)}), \tag{10}$$

where $w_{2,V}^{\bar{\nu},p}$ and $w_{2,V}^{\nu,p}$ denote the contributions of the vector interaction to $w_1^{\bar{\nu},p}$ and $w_2^{\nu,p}$ in (4), and the functions $w_2^{(1/2)}$ correspond and $w_2^{(3/2)}$ to the isovector transitions to the states with T = 1/2 and T = 3/2 in electroproduction on protons. Substituting (9) and (10) at large values of $|q^2|$, we obtain

$$\int_0^2 d\omega \left[\frac{1}{2}\left(\frac{m^2\omega}{|q^2|}\right)^{\gamma_T^a} F_T^a(\omega) + \left(\frac{m^2\omega}{|q^2|}\right)^{\gamma_L^a + 1} \omega F_L(\omega)\right] = 1. \tag{11}$$

The left side of (11) will not depend on q^2 if $\min(\gamma_T\gamma_L + 1) = 0$. Then w_1^a and w_2^a can be represented in the form $w_1^a(\nu, \omega) = 1/2)(m^2/\nu)^{\gamma_T^a} F_T^a(\omega)$ $(\gamma_T^a > 0)$ and $w_2^a(\nu,\omega) = (1/\nu) F_2^a(\omega)$. If we assume the hypothesis of Abarbanel, Goldberger, and Treiman [9] that the Regge asymptotic form describes the behavior of the scattering amplitudes also when $|q^2| \sim \nu \to \infty$, then F_2^a and F_T^a will be given by $F_2^a = C_2^a \omega^{-\alpha_a+1}$ and $F_T^a = C_T^a \omega^{-\alpha_a - \gamma_T^a}$, where C_2^a and C_T^a are constants, α_a is the position of the Regge pole with negative signature farthest to the right, and C = P = 1 at T = 0.

Since $w_1 \geq (1/2)w_1^a$ and $w_2 \geq (1/2)w_2^a$, we have

$$w_2(\nu, \omega) \geqslant \frac{1}{\nu}F_2(\omega), \qquad w_1(\nu, \omega) \geqslant F_1(\omega)\left(\frac{m^2}{\nu}\right)^{\gamma_T^a}. \tag{12}$$

An asymptotic expression of this type (with the equal sign in (12) and with $\gamma_T^a = 0$) was proposed by Bjorken [10] on the basis of entirely different considerations.

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RAMAN SCATTERING OF AN ELECTROMAGNETIC WAVE OF FREQUENCY MUCH LOWER THAN THE CHARACTERISTIC TRANSITION FREQUENCY IN SEMICONDUCTORS

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Raman scattering (RS) of light in solids have been the subject of an extensive study in recent years. In all the hitherto preformed experiments, however, the RS was observed in the optical and infrared bands (incident-radiation wavelength $\lambda_1 \lesssim 10\ \mu$, with a small frequency shift, $\Delta\omega = \omega_1 - \omega_2 << \omega_1, \omega_2$, where ω_1 and ω_2 are the frequencies of the incident and scattered radiation, respectively.

In this communication we shall consider RS in the limiting case when the frequency of the incident radiation is much lower than the scattered frequency. The scattering in semiconductors if from the Landau levels and from optical phonons; estimates show that the semiconductor n-InSb the cross section of RS from the Landau levels can reach values on the order of 10^{-10} cm^{-1} even in the millimeter band (we shall henceforth define the SR cross section as the true cross section (in cm^2) multiplied by the scatterer density).