angular momentum J and parity. All the barred quantities in (4) correspond to interference in the square of the matrix-element modulus. The summation in (4) is over the j, j', $\bar{j}$', L, and $\bar{L}$ and over the odd values of $\ell \le \min\{2j_\gamma, 2j_3\}$.

In the derivation of (4) it was assumed that the contribution of a single resonance is of importance in the scattering. If several resonant states take part in the reaction, then β in (4) must be replaced by $\bar{\beta}$, the mean value of the amplitude of the impurity states with opposite parity over a certain energy interval.

By way of an example let us consider several particular cases of formula (4). If $j_2 = j_4 = 3/2^+$, $j_3 = 5/2^+$, $\ell = L = L' = j = j_\gamma = 1$, $\bar{L} = 0$, $j' = \bar{j}' = 2$, and $J = 1^- + \beta 1^+$, then the degree of the circular polarization of the photons is given by

$$\mathcal{P} = -\frac{21\sqrt{2}}{40\sqrt{\pi}}\,\frac{\sqrt{\Gamma_{11}\Gamma_{01}}}{\Gamma_{11}+\Gamma_{01}}\cos(\delta_{11}-\delta_{01})\cos\theta .$$

In the case when $j_2 = j_4 = 1/2^+$, $j_3 = 3/2^+$, $\ell = L = j = j_\gamma = 1$, $L' = 0$, $j' = \bar{j}' = L = 2$, and $j = 2^- + \beta 2^+$, the degree of circular polarization is

$$\mathcal{P} = -\frac{3}{2}\sqrt{\frac{15}{\pi}}\,\frac{\sqrt{\Gamma_{21}\Gamma_{11}}}{\Gamma_{21}+\Gamma_{11}}\cos(\delta_{11}-\delta_{21})\cos\theta .$$

We note that a formula similar in many respects to formula (4) holds also in the case of non-resonant scattering of particles by nuclei.

To obtain a large effect of dynamic enhancement in experiment, it is convenient to use nuclei and reactions in which the level density of the excited nucleus is small, so that levels with a given spin but with different parity strongly overlap.

The authors are deeply grateful to Professor I. S. Shapiro for a discussion of the work and for many valuable remarks.

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SELECTIVE TRANSPARENCY OF FERROMAGNETIC FILMS

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Submitted 11 July 1969; resubmitted 20 August 1969
ZhETF Pis. Red. 10, No. 7, 336 - 340 (5 October 1969)

Recently B. Heinrich and V. Meshcheryakov [1] observed a previously predicted [2] selective transparency of a ferromagnetic plate (they used permalloy in their experiment) at a frequency ω satisfying the antiresonance (AR) condition

$$\omega = gB , \tag{1}$$

where $B = H + 4\pi M$ is the magnetic induction, M the magnetic moment per unit volume, H the magnetic field in the film, and g the gyromagnetic ratio. At the AR frequency, the real part of the magnetic susceptibility (μ') vanishes and the imaginary part (μ") is small if the ferromagnetic resonance (FR) line width is sufficiently small. Since the penetration skin depth is $\delta \sim \mu^{-1/2}$ ($\mu = \mu' + i\mu''$), a sharp increase of the penetration skin depth should be observed at $\omega = gB$. If we neglect μ'', then the skin depth at the AR frequency becomes infinite [2].

An experimental observation of the selective transparency [1] requires that the phenomenon be investigated more completely than in [2].

The high-frequency properties of the plate material are described by the magnetic-susceptibility tensor $\mu_{ik}(\omega)$ and by the electric resistivity σ. Since polycrystalline films were used in the experiment, in which the electron mean free path was not too large, the dispersion of σ and its dependence on the magnetic field can be neglected. The rotation of the magnetic moment under the influence of the external magnetic field leads to gyrotropy of the tensor μ_{ik}: $\mu_{ik} = \mu^{*}_{ki}$ (we neglect magnetic dissipation); for a polycrystal $\mu = \mu_{22}$ (the "3" axis is along the equilibrium moment).

If the wave propagation direction z makes an angle ν with the direction of the equilibrium magnetic moment, than the two waves propagating in the ferromagnet are elliptically polarized, and the high-frequency components of the magnetic field and of the induction are connected by the following relations:

$$B_{\pm} = \mu_{\pm} H_{\pm}\,; \qquad H_{\pm} = H_x + ip_{\pm} H_y \quad , \tag{2}$$

where $E_{\pm} = E_y - ip_{\pm}E_x$. The quantities $\mu_{\pm}$ and $p_{\pm}$ are expressed in terms of the components of the tensor μ_{ik} (we shall not write out the rather cumbersome expressions). It is easy to verify that only one of the quantities, μ_+ or μ_-, vanishes. We shall henceforth denote it simply by μ. Near AR we have

$$\mu \approx \frac{\omega - gB}{2\pi g M(1 + \cos^2\nu)} \quad , \tag{3}$$

and the corresponding polarization at the AR point is

$$p = 1/\cos\nu. \tag{4}$$

The wave is circularly polarized when $\nu = 0$, and linearly when $\nu = \pi/2$.

We consider now the polarized wave (with polarization equal to p) incident on the ferromagnetic film of thickness d. Recognizing that the square of the wave vector is

$$k^2 = \frac{\omega^2}{c^2}\,\frac{4\pi\sigma}{\omega}\,\mu(\omega)\,i = \frac{2\sigma g B}{c^2}\,\frac{B}{(1+\cos^2\nu)M}\,i\eta\,, \tag{5}$$

where $\eta = (\omega - gB)/gB$ is the deviation from the AR, we can readily find that near the AR ($|\eta| \ll 1$) the transparency coefficient ξ (the ratio of the energy flux density in the transmitted wave to that in the incident wave) equals

$$\xi = \frac{c^2}{4\pi\sigma^2 d^2}\,\frac{2x^2}{\sin^2 x + sh^2 x} \quad , \tag{6}$$

where

$$x = \left[\frac{\sigma g B}{c^2} \quad \frac{B \mid \eta \mid}{M(1 + \cos^2 \nu)} \right]^{1/2} d \; . \tag{7}$$

The obtained formulas (6) and (7) show that the transmission line width $\xi(x)$ is inversely proportional to the square of the film thickness

$$\frac{\Delta\omega}{\omega_{AR}} \sim \left(\frac{\delta_o}{d}\right)^2; \quad \delta_o \sim \frac{c}{\sqrt{\sigma\omega_{AR}}} \quad ; \qquad \omega_{AR} = gB . \tag{8}$$

We recall that $d \gg \delta_0$.

The transmission line has a complicated structure. When $|x| \ll 1$ we have

$$\xi/\xi_{max} = 1 - \frac{13}{3 \cdot 5!} x^y , \tag{9}$$

i.e., $(\xi_{max} - \xi)/\xi_{max} \sim (\Delta\omega)^2$ (see (7) and (5)). When $|x| \gg 1$, the transparency coefficient decreases "almost: exponentially

$$\xi \approx 4 \xi_{max} x^2 e^{-2x} . \tag{10}$$

When using formulas (6) and (7), it must be recalled that the transparency coefficient is expressed in terms of the values of H and ν inside the plate.

In the derivation of formula (6) we did not take into account: a) the imaginary part of the magnetic susceptibility (μ'') and b) its spatial dispersion.

a) Allowance for μ'' leads to an insignificant lowering of the transparency coefficient and changes slightly the line shape near the maximum (at $\Delta\omega \sim 1/\tau_M$, where τ_M is the magnetic-moment relaxation time, $1/\tau_M$ is of the order of the FR line width).

b) The role of spatial dispersion in AR will be the suject of a separate communication. We shall make here only a few remarks. The magnetic susceptibility with allowance for spatial dispersion has the following form [3]:

$$\mu = \frac{\omega - \omega_{AR} - a' k^2}{\omega - \omega_{FR} - a k^2} , \tag{11}$$

k is the wave vector, $\omega_{AR} = gB$, and ω_{FR} is the FR frequency, which depends on the wave propagation direction (if $\nu = 0$, then $\omega_{FR} = gH$, and if $\nu = \pi/2$, then $\omega_{FR} - \dot{g}\sqrt{HB}$, $\omega_{FR} < \omega_{AR}$), the exchange constants α and α' have an order of magnitude $\theta_C a^2/\hbar$, where θ_C is the Curie temperature (in the general case $\alpha' \neq \alpha$, and only when $\nu = 0$ do we have $\alpha' = \alpha$). An analysis of the dispersion equation

$$k^2 = \frac{\omega^2}{c^2} \frac{4\pi i\sigma}{\omega} \mu(\omega, k)$$

shows that when $\omega = \omega_{AR}$ there exists a zero solution ($k = 0$, $\delta = \infty$) also in this case. Allowance for the spatial dispersion does not change the conclusion that the film has selective transparency at AR. However, near $\omega = \omega_{AR}$, besides the wave with small wave vector (that vanishes at the AR), there exists also another wave (spin wave), whose wave vector k_s has, according to (11), an order of magnitude (when $\omega = \omega_{AR}$ and when $\delta_0 \gg a\sqrt{\theta_C/\hbar\tilde{\omega}}$)

$$k_s = \frac{1}{a}\left(\frac{\hbar\tilde{\omega}}{\theta_c}\right)^{1/2} + \frac{1}{\delta_0}\frac{a}{\delta_0}\left(\frac{\theta_c}{\hbar\tilde{\omega}}\right)^{1/2} i \qquad (12)$$

($\tilde{\omega} = \omega_{AR} - \omega_{FR}$). If the film thickness d is small compared with the attenuation wave of the spin wave ($d << (\delta_0^2/a)(\hbar\tilde{\omega}/\theta_C)^{1/2}$), then the transparency coefficient can vary appreciably at a fixed ratio of the spin wavelength to the film thickness. The result depends on the boundary conditions from the magnetic moment, i.e., essentially on the character of the film surface. When $d \geq [(\delta_0^2/a)(\hbar\tilde{\omega}/\theta_C)^{1/2}]$, the role of the spin wave in the AR is small, and only a small fraction of the electromagnetic-wave energy goes to excitation of the spin wave, which attenuates in the film. This decrease somewhat the transparency coefficient (6). A numerical comparison of formula (6) with the results of Heinrich and Meshcheryakov is difficult, since the absolute value of the transmission coefficient is not given in [1]. In addition, the formula derived here is applicable to a small vicinity of $\omega = \omega_{AR}$, and can therefore not take into account the experimentally observed asymmetry of the transmission line. A qualitative comparison shows the theory to agree with experiment. In particular, the line width is inversely proportional to the square of the film thickness.

I take the opportunity to thank B. Henrich and V. Meshcheryakov for reporting their results prior to publication, and also A. S. Borovik-Romanov, B. Heinrich, and R. P. Yankelevich for useful discussions.

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AGREEMENT BETWEEN THE EXPERIMENTAL DATA DESCRIBING THE ELECTROMAGNETIC FORM FACTOR OF THE PION

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Submitted 24 July 1969
ZhETF Pis. Red. 10, No. 7, 340 - 344 (5 October 1969)

1. Investigation of the reaction $e^+e^- \rightarrow \pi^+\pi^-$ with the aid of colliding-beam experiments have yielded direct information concerning the π-meson electromagnetic form factor G(t) in the timelike region $t \geq t_0 = 4m_\pi^2$ [1]. Information concerning G(t) in the spacelike region t < 0 was obtained by several methods, particularly by the Chew-Low extrapolation from the pion electroproduction reaction $e^-p \rightarrow e^-\pi^+n$ [2]. A method for determining the mean-squared pion radius r_π, based on the analytic properties of G(t), was proposed in [3]. This method possesses the following features: a) use is made of experimental information on G(t) for both $t > t_0$ and t < 0, in the finite interval $[t_2, t_1]$, $t_2 < t_1 < 0$; b) it becomes unnecessary to extrapolate G(t) directly from the t < 0 side to the point t = 0 (this is important, for when t is small the value of G(t) is subject to large experimental errors); c) by making certain assumptions concerning G(t), it is possible to conclude that the experimental values of G(t) at $t > t_0$ and t < 0 are analytically compatible; d) in the particular case when $t_2 \rightarrow t_1$ and