$$k_s = \frac{1}{a}\left(\frac{\hbar\tilde{\omega}}{\theta_c}\right)^{1/2} + \frac{1}{\delta_0}\,\frac{a}{\delta_0}\left(\frac{\theta_c}{\hbar\tilde{\omega}}\right)^{1/2} i \qquad (12)$$

($\tilde{\omega} = \omega_{AR} - \omega_{FR}$). If the film thickness d is small compared with the attenuation wave of the spin wave ($d \ll (\delta_0^2/a)(\hbar\tilde{\omega}/\theta_C)^{1/2}$), then the transparency coefficient can vary appreciably at a fixed ratio of the spin wavelength to the film thickness. The result depends on the boundary conditions from the magnetic moment, i.e., essentially on the character of the film surface. When $d \gtrsim [(\delta_0^2/a)(\hbar\tilde{\omega}/\theta_C)^{1/2}]$, the role of the spin wave in the AR is small, and only a small fraction of the electromagnetic-wave energy goes to excitation of the spin wave, which attenuates in the film. This decrease somewhat the transparency coefficient (6). A numerical comparison of formula (6) with the results of Heinrich and Meshcheryakov is difficult, since the absolute value of the transmission coefficient is not given in [1]. In addition, the formula derived here is applicable to a small vicinity of $\omega = \omega_{AR}$, and can therefore not take into account the experimentally observed asymmetry of the transmission line. A qualitative comparison shows the theory to agree with experiment. In particular, the line width is inversely proportional to the square of the film thickness.

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AGREEMENT BETWEEN THE EXPERIMENTAL DATA DESCRIBING THE ELECTROMAGNETIC FORM FACTOR OF THE PION

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1. Investigation of the reaction $e^+e^- \to \pi^+\pi^-$ with the aid of colliding-beam experiments have yielded direct information concerning the π-meson electromagnetic form factor G(t) in the timelike region $t \geq t_0 = 4m_\pi^2$ [1]. Information concerning G(t) in the spacelike region $t < 0$ was obtained by several methods, particularly by the Chew-Low extrapolation from the pion electroproduction reaction $e^-p \to e^-\pi^+n$ [2]. A method for determining the mean-squared pion radius r_π, based on the analytic properties of $\Gamma(t)$, was proposed in [3]. This method possesses the following features: a) use is made of experimental information on G(t) for both $t > t_0$ and $t < 0$, in the finite interval $[t_2, t_1]$, $t_2 < t_1 < 0$; b) it becomes unnecessary to extrapolate G(t) directly from the $t < 0$ side to the point $t = 0$ (this is important, for when t is small the value of G(t) is subject to large experimental errors); c) by making certain assumptions concerning G(t), it is possible to conclude that the experimental values of G(t) at $t > t_0$ and $t < 0$ are analytically compatible; d) in the particular case when $t_2 \to t_1$ and

$t_1 \to 0$ the method is equivalent to the method used to calculate r_π, in which information on $G(t)$ is used only for $t > t_0$ (cf., e.g., [4]). In concrete calculations of r_π made in [3], the results of [1] and [2] were used. Recently, however, new experimental data on $G(t)$ were published for both $t > t_0$ [5, 6] and $t < 0$ [7]. The Novosibirsk group [5] obtained for the ρ-meson peak width Γ a value larger than expected from [1] (Γ = 105 ± 20 meV) and in satisfactory agreement with the measurements of the Orsay group [6] (Γ = 112 ± 12 meV). On the other hand, the discrepancy between the results obtained by these groups for $|G(t = m_\rho^2)|$ is still quite large. The purpose of the present communication is to check the analytic consistency of $G(t)$ and to determine the range of variation of r_π in light of the present experimental data.

To calculate r_π, we shall use the dispersion relation (1) from [3][1)]

$$\ln G(t) = \frac{1}{\pi}\sqrt{\frac{(t-t_1)(t-t_2)}{t_0-t}}\left\{\int_{t_0}^{\infty}\sqrt{\frac{t'-t_0}{(t'-t_1)(t'-t_2)}}\,\frac{\ln|G(t')|\,dt'}{t'-t} + \int_{t_2}^{t_1}\sqrt{\frac{t_0-t'}{(t_1-t')(t'-t_2)}}\,\frac{\ln G(t')}{t'-t}\,dt'\right\}. \tag{1}$$

To describe the experimental data, we shall use the following formulas: when $t > t_0$

$$|G(t)|^2 = km_\rho^2\Gamma^2[(m_\rho - t)^2 + m_\rho^2\Gamma^2]^{-1}, \tag{2}$$

where, according to [6], k = 55.6 ± 6.2, Γ = 112 ± 11.5 meV, m_ρ = 760 ± 5.5 meV, and when $t_2 < t < t_1 < 0$

$$G(t) = (1 - t/m^2)^{-1}, \tag{3}$$

where according to [7] m = 560 ± 80 meV, corresponding to r = 0.86 ± 0.14 F when (3) is directly extrapolated to the point t = 0.

3. Calculations show that the uncertainty in the parameter m_ρ does not change r_π by more than 0.03 F (the results that follow were determined for $t_1 = -0.34$ GeV^2. However, the dependence of r_π on the parameters k, Γ, and m is strong. Some examples of such a dependence are shown in Fig. 1. Thus, when Γ is varied from $\Gamma_{(-)} \sim 100$ meV to $\Gamma_{(+)} \sim 125$ meV and the other parameters remain unchanged, r_π (t_2) fills the region bounded by curves 1 and 2; curves 1 and 3 limit the region filled by r_π (t_2) when m is varied from $m_{(+)}$ to $m_{(-)}$, and curves 1 and 4 bound the region of variation of k from $k_{(-)}$ to $k_{(+)}$. The results of the calculation can be formulated as follows: a) the set of parameters ($k_{(0)}$, m_ρ, $\Gamma_{(0)}$, $m_{(0)}$ is not compatible analytically (see curve 5 of Fig. 1), since r_π (t_2) decreases strongly with increasing $(-t_2)$, and becomes negative when $t_2 \geq t_m$ (t_m is the smallest t < 0 for which experimental data

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1) Here $t \in [t_1, t_0]$; it is assumed that G(t) has no complex zeroes. The following misprints should be corrected in [3]: insert ln in front of G(t) in formula (1), insert m_ρ after Γ_ρ in (5), and replace m^{-2} by m^2 in (4).

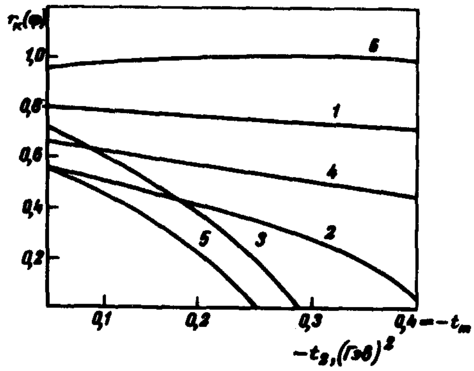

Fig. 1. Values of r_π for the single-resonance case; curve 1 is for the set $k_{(-)}, m_{\rho(0)}, \Gamma_{(-)}, m_{(+)}$, 2 – $(k_{(-)}, m_{\rho(0)}, \Gamma_{(+)}, m_{(+)})$, 3 – $(k_{(-)}, m_{\rho(0)}, \Gamma_{(-)}, m_{(-)})$, 4 – $(k_{(+)}, m_{\rho(0)}, \Gamma_{(-)}, m_{(+)})$, 5 – $(k_{(0)}, m_{\rho(0)}, \Gamma_{(0)}, m_{(0)})$, 6 – $(k = 35{,}5, m_{\rho(0)}, \Gamma_{(-)}, m_{(+)})$

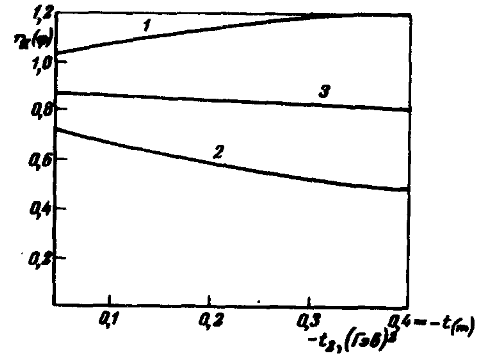

Fig. 2. Values of r_π for the two-resonance modification of (4). The curve 1 – set $(k_{(-)}, m_{\rho(0)}, \Gamma_{(-)}, m_{(+)})$, 2 – $(k_{(+)}, m_{\rho(0)}, \Gamma_{(+)}, m_{(0)})$, 3 – $(k_{(0)}, m_{\rho(0)}, \Gamma_{(0)}, m_{(0)})$

on G(t) are available [7]; b) when $t_2 = t_m$ we have $r < 0.73$ F for all possible sets; c) the incompatibility of the present data does not make it possible to indicate an upper bound for r_π; d) a tendency towards compatibility appears if $k \to k_{(-)}$, $\Gamma \to \Gamma_{(-)}$, and $m \to m_{(+)}$; e) the only relatively compatible set is $k_{(-)}, m_\rho, \Gamma_{(-)}, m_{(+)}$ (see curve 1 of Fig. 1). Thus, if we assume that G(t) has no complex zeroes and is well represented at $t > t_0$ by a single-resonance formula, then the tendency of the data for $k \to k_{(-)}$ and $\Gamma \to \Gamma_{(-)}$ to be compatible apparently indicates that preference should be given to the ρ-resonance parameters determined in [1, 5] (curve 6 of Fig. 1 was determined for k = 35.5, which is within the limits of the measurement error of $|G(m^2_\rho)|$ [1, 5]).

4. The conclusion that the data of [1, 5] should be given preference depends strongly on the behavior of G(t) when $t > m^2_\rho$, since the decrease of the intergrand of the first integral of (1) as $t \to +\infty$ is quite slow. By modifying the asymptotic form of G(t) by including the hypothetical ρ' meson, in the same manner as in [3], i.e., when $t > t_0$

$$G(t) = \sqrt{k}\, m_\rho \Gamma \Big\{ [m_\rho - t - i\Gamma m_\rho (t - t_0)^{1/2} / (m^2_\rho - t_0)^{1/2}]^{-1} - -[m^2_{\rho'} - t - i\Gamma_{\rho'}(t - t_0)^{1/2}]^{-1} \Big\}, \tag{4}$$

where $m_{\rho'} = 1.85$ GeV, $\Gamma_{\rho'} = 0.15$ GeV, and k, Γ, and m_ρ are taken from [6], we obtain the results shown in Fig. 2. The sets of parameters containing $m = m_{(-)}$ turn out to be poorly compatible, as in the single-resonance approximation, and are not shown in Fig. 2. The

$r_\pi(t_2)$ corresponding to the remaining variants lie inside the region bounded by curves 1 and 2. It is interesting to note that the set $k_{(0)}$, m_ρ, $\Gamma_{(0)}$, $m_{(0)}$ (see curve 3 of Fig. 2) turns out in this case to be quite compatible, and the set $k_{(-)}$, m_ρ, $\Gamma_{(-)}$, $m_{(+)}$, as in the single-resonance approximation, yields the largest values of $r_\pi(t_2)$ (see curves 1 of Figs. 1 and 2). Consequently, if further experiments confirm the data [6] concerning the width and the height of the ρ peak, then, from this point of view, this will be a strong argument favoring the existence of a heavy ρ meson (of course, if the assumption that there are no complex zeroes of G(t) is valid.

5. For a further clarification of the situation it would be desirable to obtain more accurate data on $|G(m_\rho^2)|$ and Γ, and also additional information on G(t) at $t \sim -0.12$ GeV^2, since the results of [2], and [7], pertaining to this point are essentially different.

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VECTOR DOMINANCE AND PRODUCTION OF $\pi^\pm$ MESONS BY POLARIZED PHOTONS AT HIGH ENERGIES

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The connection between pion photoproduction on nucleons and the production of vector mesons in πN collisions has been under intense investigation of late on the basis of the vector dominance model (VDM).

The VDM predictions for $\pi^\pm$-meson production by polarized photons are usually written, excluding the ωρ interference and neglecting the φ-meson contribution, in the form [1, 2]:

$$A = \frac{\sigma_\perp^+ + \sigma_\perp^- - \sigma_\parallel^+ - \sigma_\parallel^-}{\sigma_\perp^+ + \sigma_\perp^- + \sigma_\parallel^+ + \sigma_\parallel^-} = \frac{g_{\gamma\rho}^2\,\sigma_{1-1}^\rho + g_{\gamma\omega}^2\,\sigma_{1-1}^\omega}{g_{\gamma\rho}^2\,\sigma_{11}^\rho + g_{\gamma\omega}^2\,\sigma_{11}^\omega} \approx \frac{\sigma_{1-1}^\rho}{\sigma_{11}^\rho} = \frac{\rho_{1-1}^\rho}{\rho_{11}^\rho}, \qquad (1)$$

where $\sigma_\perp^\pm$ and $\sigma_\parallel^\pm$ are the differential cross section for $\pi^\pm$-meson photoproduction by photons that are polarized perpendicular and parallel to the reaction plane, respectively; $2\sigma^\pm = \sigma_\perp^\pm + \sigma_\parallel^\pm$; $\sigma_{ij}^v = \sigma_{ij}^v\sigma^v$, where σ_{ij}^v are the elements of the spin density matrix of the vector meson, σ^v is the differential cross section of the process $\pi^- p \to V^0 n$, and $g_{\gamma V}$ is the photon to vector meson transition constant.

The contribution made by the ω meson to A amounts to several per cent [3]. We shall henceforth take into account only the ρ-meson contribution and omit the index v of ρ_{ij}^v.

It was observed in [4] that relation (1) is strongly violated in the c.m.s. However, the VDM predictions formulated in the language of helicity amplitudes are not relativistically invariant and admit of an ambiguous interpretation. The right side of (1) depends on the relativistic transformations of the reference system in the reaction plane, while the left