Table II).

Table II

E, eV	$10^{14}\div10^{15}$	10^{16}	$5\cdot10^{16}$	$5\cdot10^{17}$	10^{18}	10^{19}
δ, %	0.1 [24]	0.7 [10]	3.0 [10]	3.4 [23]	10 [11]	30 [11]

Table II lists the upper limits of the variation amplitudes, for which only the orders of magnitude are given. At lower energies, when the main role is played by the stationary component, $\delta \approx (0.01 - 1)\%$ at the chosen model parameter, and there is no contradiction with the data of Table II within the limits of the experimental errors.

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INFLUENCE OF TEMPERATURE PERTURBATIONS ON PLASMA DIFFUSION IN TOROIDAL SYSTEMS

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The success of the "Tokamak" program [1], which has made it possible to reduce the particle loss from the trap to the level of the classical loss due to pair collisions [2], has attracted the interest of specialists to the review, previously initiated by us, of the theory of transport phenomena in a rarefied plasma in a toroidal system [3]. Quite recently,

T. Stringer [4] undertook to improve also on the Pfirsch-Schluter theory [5], which was developed for the dense-plasma limit. He noted that in a coordinate system rotating at the speed of the electric plasma drift, the toroidal magnetic field acts like a driving force with spatial and temporal periods corresponding to the mode $m = 1$ with longitudinal wave number $k = \theta/r$ and with frequency $\omega_0 = - v_0/r$ (θ - angle of rotation transformation angle, v_0 - electric drift velocity). The response to this action is proportional to the dielectric constant of the plasma $D(- v_0/r; \theta/r)$ and increases rapidly if resonance is established between the driving force and the drift oscillations of the inhomogeneous plasma. The diffusion coefficient is much higher in this case than that obtained by Pfirsch and Schluter.

M. Rosenbluth and J. Taylor [6] indicated later that in T. Stringer's model there is no quasistationary state of the plasma, since the non-ambipolar diffusion leads at all time to an increase of the electric field and to an untwisting of the plasma. To eliminate this shortcoming, they took into account the viscosity of the ionic plasma component on the diffusion. Since the influence of low viscosity becomes appreciable only when the external force is in resonance with the natural plasma oscillations, the rotary velocity of the plasma in the quasistationary state turned out to be very high (of the order of the phase velocity of sound along the small circuit of the torus):

$$v_0 = \pm \theta c_s, \quad r_{ci}/\theta r << 1, \tag{1}$$

where $c_s = [(T_{0i} + T_{0e})/m_i]^{1/2}$ is the speed of sound and r_{ci} is the Larmor radius of the ion.

The plasma diffusion coefficient in such a state turned out to be so high, that a thorough review was necessary of all the presently available data (thus, for the Munich stellarator, the diffusion time calculated in accordance with [6] was shorter than the experimentally measured value).

We shall show in this article that although the Stringer effect remains in force, nonetheless the equilibrium electric drift velocity is always far from resonance. The plasma diffusion in the stable regime has therefore the same order of magnitude as that obtained by Pfirsch and Schluter. The reason for such an appreciable change of the results of Rosenbluth and Taylor is a correct allowance for the temperature perturbations and the thermal forces associated with them, since, as was shown by us earlier [7, 8], the thermal force and the electron-ion friction enter additively into the increment of the drift instability in the limit when the plasma has good conductivity.

By way of the simplest model of a magnetic trap, we consider the following magnetic-field configuration

$$\mathbf{H} = H_0 \{\mathbf{e}_z + \theta\, \mathbf{e}_\nu\},$$
$$\theta = (i\,\epsilon/2\pi) << 1, \quad \epsilon = (r/R) << 1. \tag{2}$$

We imitate the toroidal character of the system by a gravitational field

$$\mathbf{g}_i = (2T_i/m_i R)\{\mathbf{e}_r \cos\nu - \mathbf{e}_\nu \sin\nu\}. \tag{3}$$

To solve the two-fluid magnetohydrodynamic equations describing the plasma in the model (2) - (3), we use, following Stringer [4], the method of expanding in the toroidality parameter. Unlike in [4 - 6], however, we take into account the fact (known from stability theory [7]) that the correction to the particle temperature T_1 turns out to be inhomogeneous in space even if there is no gradient of the unperturbed temperature ($T_0(r) = \text{const}$). Therefore we add to the generalized Ohm's law (see Eq. (8) of [4]) the force

$$F = -(1 + s)\, n_0 \nabla_{\parallel} T_{e1}; \quad s = 0{,}71, \tag{4}$$

and to find the correction to the temperature we used the linearized thermal-conductivity equation [9]

$$- T_{0e}\{ (v_0 + U_{ne}) \frac{\partial}{r\,\partial\nu} [n_1 - 2\epsilon n_0 \cos\nu] + v_{r1} \frac{\partial n_0}{\partial r}\} = \theta^2 \kappa_{\parallel}^{e} \frac{\partial^2}{r^2 \partial\nu^2} T_{e1} - \frac{s\,\theta T_{0e}}{e} \frac{\partial j_{\parallel}}{r\,\partial\nu}, \tag{5}$$

where

$$U_{nj} = \frac{c T_{0j}}{e H_0 n_0} \frac{dn_0}{dr}; \quad v_0 = \frac{c}{H_0} \frac{d\Phi_0}{dr};$$

and $j_{\parallel}$ is the current density.

Following Stringer [4], we obtain the ion flux across the magnetic field

$$\langle n v_r \rangle_i = D_{PS} \frac{dn_0}{dr} \frac{\theta^2 c_s^2}{D(v_0)} \left[1 + \frac{s^2}{\alpha} + \frac{s\, v_0 (v_0 + U_{ni})(v_0 + U_{ne})}{\alpha D (U_{ni} - U_{ne})} \right], \tag{6}$$

where

$$D_{PS} = \xi_e r_{ce}^2 \frac{4\pi^2}{i^2} (1 + \frac{T_{0i}}{T_{0e}})$$

is the Pfirsch-Schluter diffusion coefficient; $D(v_0) = v_0^2 + v_0 U_{ne} - \theta^2 c_s^2$ is a quantity proportional to the dielectric constant; $\alpha \equiv e^2 \eta_{\parallel} \kappa_{\parallel}^{e} / T_{0e} \approx 2.26$ is a parameter ($\kappa_{\parallel}^{e}$ is the thermal-conductivity coefficient and $\eta_{\parallel}$ is the plasma resistance).

In the general case the diffusion is not ambipolar and the plasma untwists to a stationary velocity v_0. We obtain the equation for $v_0(t)$ from the plasma motion equation with allowance for the quasineutrality condition [6]. In our case it takes the form

$$n_0 m_i \frac{\partial v_0}{\partial t} = \frac{2\epsilon^2}{\theta^2} \frac{n_0 T_{0e}}{\kappa_{\parallel}^{e}} \frac{v_0}{D^2(v_0)} \{ [v_0 + U_{ni} + s(U_{ni} - U_{ne})] \times [v_0 (v_0 + U_{ni})(v_0 + U_{ne}) + s D(v_0)(U_{ni} - U_{ne})] + \alpha D(v_0)(U_{ni} - U_{ne})^2 \} \tag{7}$$

We see thus that stable stationary rotation is attained only at one velocity

$$v_0 \approx -U_{ni} [1 + s(1 + \frac{\alpha}{s^2})(1 + \frac{T_{0e}}{T_{0i}})]; \quad \frac{U_{ni} - U_{ne}}{\theta c_s} << 1, \tag{8}$$

and the diffusion coefficient differs from D_{PS} by a factor[1)]

$$G = 1 + s^2/a = 1.31. \tag{9}$$

It is important to note that even in the limit of strong gradients ($U_{ni} > \theta C_s$) the plasma rotation frequency, in accord with the condition $\partial v_0/\partial t = 0$, is always far from resonance and therefore the diffusion coefficient does not exceed D_{PS} significantly.

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ERRATUM

In the article by V. I. Mel'nikov and E. I. Rashba, Vol. 10, No. 2, p. 61, formula (9) should read:

$$\frac{\omega^2 - \Omega^2}{16\pi\omega^2} = \max_f \{ \int d\mathbf{R}\, \psi_0^2 (\nabla f)^2 / \int d\mathbf{R} [\nabla(\Delta f + 2\nabla f \nabla \ln \psi_0)]^2 \}. \tag{9}$$

[1)] The plasma turns out stable in this case, owing to the stabilizing influence of the longitudinal ion inertia.