processes (2), inasmuch as in the former case the corresponding photonless processes is one of the main decay channels.

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INSTABILITY OF OSCILLATIONS WITH FREQUENCY EQUAL TO HALF THE ION-CYCLOTRON FREQUENCY

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Submitted 18 August 1969
ZhETF Pis. Red. 10, No. 8, 389 - 392 (20 October 1969)

1. It is known that in adiabatic traps the plasma can be unstable against the buildup of ion-cyclotron oscillations, i.e., oscillations with frequencies close to the ion-cyclotron frequency ω_i or to its harmonics $n\omega_i$. However, buildup of oscillations with frequencies that were multiples of half the cyclotron frequency, $\omega = n\omega_i/2$, was observed in experiments with the Phoenix apparatus [1] and with Ogra-1 (unpublished) in addition to the ion-cyclotron instability. In our opinion, the excitation of such oscillations may be connected with parametric resonance. Let us explain this statement. The ion-cyclotron instability begins to build up at a plasma density such that the frequency of the magnetized electronic Langmuir oscillations becomes comparable with the ion-cyclotron frequency [2, 3]. In a homogeneous and unbounded plasma, the spectrum of the frequencies of the magnetized electron Langmuir oscillations is given by the formula $\omega = \omega_{pe}k_{\parallel}/k \leq \omega_{pe}$, where ω_{pe} is the electron Langmuir frequency and $k_{\parallel}$ is the component of the wave vector along the magnetic field. In bounded systems, the spectrum is discrete and in order for the oscillation to build up it is necessary that the maximum frequency in this spectrum be comparable with ω_i. (This frequency is smaller than ω_{pe}, since oscillations with $k_{\perp} \neq 0$ are built up). The oscillations with lower frequencies remain stable within the framework of the linear theory. In the presence of instability at the ion-cyclotron frequency, however, the plasma parameters become alternating in time, and this creates conditions for parametric buildup of oscillations with $\omega = \omega_i/2$. At much higher values of the density, $\omega_{pe} > n\omega_i$ and parametric buildup of oscillations with $\omega \approx \ell\omega_i/2$ becomes possible ($\ell = 1, 2, \ldots, 2n$).

2. Let us assume that oscillations of the electric potential are excited in an adiabatic trap at the ion-cyclotron frequency. The systems of interest to us (Phoenix, Ogra-1) have axial symmetry (the symmetry axis is parallel to the magnetic field and to the OZ axis). Therefore, the perturbations of the potential will be of the form $\phi_1(r, t) = \phi_1(r,z)\cos(\omega_i t - n_1\theta)$, where θ is the azimuthal angle. In the analysis of the evolution of the oscillations with $\omega \approx \omega_i/2$, we shall use asymptotic methods developed in the theory of the

nonlinear oscillator [4]. In many cases these methods were used also in plasma oscillation theory [5, 6]. Following [4 - 6] we shall seek perturbations of the potential in the form $\phi_2(\vec{r}, t) = \phi_2(r, z, t)\cos(\omega_i t/2 + \psi(t) - m_2\theta)$. We shall assume henceforth that the characteristic time scale of the amplitude ϕ_2 (r, z, t), and also of the slowly-varying part of the phase $\psi(t)$ is large compared with the cyclotron period.

The time dependence of $\phi_2(r, z, t)$ and $\psi(t)$ should be determined with the aid of the equation of motion of the electrons, the continuity equation, and the Poisson equation. Simple manipulations yield

$$\dot{\psi} = \delta\omega - A\cos 2\psi, \quad (1) \qquad \dot{p} = -pA\sin 2\psi, \quad (2)$$

$$p = [\langle(\partial\phi_2/\partial z)^2\rangle]^{1/2}, \quad \delta\omega = \omega_2 - \omega_i/2,$$

ω_2 is the frequency of the oscillations under consideration in the linear approximation, i.e., at $\phi_1 = 0$.

$$A = \frac{1}{8}\,\omega_i \left\langle \frac{n_1(r,z)}{n_o(r,z)}\left(\frac{\partial\phi_2}{\partial z}\right)^2\right\rangle \left\langle\left(\frac{\partial\phi_2}{\partial z}\right)^2\right\rangle^{-1},$$

$$n_1(r,z) = \frac{e}{m}\, n_o\omega_i^{-2}\,\frac{\partial^2\phi_1}{\partial z^2},$$

n_0 is the unperturbed plasma density, and the angle brackets denote integration with respect to r and z.

It is interesting to note that precisely the same equations are obtained for p(t) and $\psi(t)$ from the Mathieu equation

$$\ddot{x} + \omega_2^2\left(1 - \frac{4A}{\omega_2}\cos\omega_i t\right)x = 0, \tag{3}$$

if the solution (3) is sought in the form [4]

$$x = p(t)\cos\left(\frac{\omega_i}{2}t + \psi(t)\right).$$

This analogy enables us to transfer the results of the investigation of (3) to our case. In particular, if the frequency of the natural oscillations ω_2 turns out to be close to $\omega_i/2$, then the oscillations under consideration will be stable. This buildup condition is given by

$$A = \frac{1}{4}\,\omega_i\,\frac{\left\langle \frac{n_1}{n_o}\left(\frac{\partial\phi_2}{\partial z}\right)^2\right\rangle}{\left\langle\left(\frac{\partial\phi_2}{\partial z}\right)^2\right\rangle} > |\delta\omega| = \left|\omega_2 - \frac{\omega_i}{2}\right|. \tag{4}$$

For the buildup to occur, it is also necessary that the azimuthal wave number m also be divided together with the frequency, i.e., that the relation $m_2 = m_1/2$ be obtained.

To verify whether condition (4) is satisfied in real systems, it is necessary to solve exactly the problem of the natural plasma oscillations in the linear approximation, with a determination of the form of the functions $n_1(r, z)$ and $\phi_2(r, z)$, and also the natural frequency ω_2. To this end it is necessary, in turn, to know the unperturbed plasma density distribution $n_0(r, z)$. However, there are no corresponding experimental data. Therefore, strictly speaking, our statement that the buildup of the oscillations with $\omega \approx \omega_i/2$ has a parametric character is only a hypothesis. It is favored by measurements of the azimuthal dependence of the perturbations [1]. From the results of these measurements it can be concluded that the oscillations with $\omega \approx \omega_i/2$ have half the amplitude wave number of the oscillations with $\omega \approx \omega_i$, as follows from the theory. No corresponding measurements were made on Ogra-1. In these experiments, excitation of oscillations with $\omega \approx n\omega_i/2$ ($7 \geq n \geq 2$) at a density much higher than critical. It is natural to attribute their buildup to the excitation of high resonances, see [4].

In conclusion we note that parametric buildup of oscillations can be regarded as a particular case of decay instability. As is well known, the condition for the decay of an oscillation with frequency ω into two oscillations with frequencies ω_1 and ω_2, is of the form $\omega = \omega_1 + \omega_2$. In the parametric resonance, we have $\omega_1 = \omega_2 = \omega/2$. On the other hand, it is shown in [6] that in a certain sense the decay instability itself is equivalent to parametric resonance. In particular, by a suitable choice of variables, the equations describing the decay instability can be reduced to the form (1) and (2). Thus, whereas in a homogeneous and unbounded plasma the magnetized Langmuir oscillations with frequency ω and wave vector $\vec{k}$, namely $\phi(r, t) = \phi \cos(\vec{k}\cdot\vec{r} - \omega t)$, break up into two other oscillations

$$\phi^{(1,2)}(r,t) = \phi^{(1,2)}(t)\cos(k_{1,2}r - \omega_{1,2}t - \psi_{1,2}(t)),$$

then in our case it is necessary to put in equations (1) and (2)

$$p = (\phi_1\phi_2)^{1/2}; \quad \psi = \frac{1}{2}(\psi_1(t) + \psi_2(t) + \delta\omega t), \ \delta\omega = \omega_1 + \omega_2 - \omega,$$

$$A = -\frac{n}{n_o} \quad \frac{\omega}{4k_\parallel}(\omega_1\omega_2)^{1/2}\left(\frac{\kappa_\parallel}{\omega} + \frac{k_{\parallel 1}}{\omega_1} + \frac{k_{\parallel 2}}{\omega_2}\right),$$

$$n = -\frac{k_\parallel^2}{\omega^2} n_o\phi \ , \qquad \phi^{(1)}/\phi^{(2)} = \frac{k_{\parallel 2}}{k_{\parallel 1}}\left(\frac{\omega_1}{\omega_2}\right)^{3/2}.$$

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