that clusters of galaxies, except those with the larger scales, represent, on the average, gravitationally coupled systems, of the supercluster type, for which the differential Hubble velocity is suppressed only partially. In such large scales, both the spectrum and the spatial correlation of the velocities are "frozen" making it possible, in principle, to obtain information concerning the correlation properties of the primordial eddies. In smaller scales, traces of these properties have been probably retained in uncondensed clouds of intergalactic gas, which are in situation I.

A confirmation of the relations discussed above yields weighty arguments in favor of the hypothesis that premordial cosmological turbulence exists. Further clarification of its properties will be made possible by more detailed observational data.

[1] L. M. Ozernoi, A. D. Chernin, ZhETF Pis. Red. 7, 436 (1968) [JETP Lett. 7, 342 (1968)]; Astron. zhur. 44, 1131 (1967) and 45, 1137 (1968) [Sov. Astronomy AJ 11, 907 (1968) and 12, 901 (1969)].
[2] Ya. B. Zel'dovich, Advances Astron. and Astrophys. 3, 241 (1965).
[3] I. D. Karachentsev, Sookshch. Byurakanskoi observ. (Comm. of the Byurakan Observatory), 39, 96 (1967).
[4] L. M. Ozernoi, Astron. tsirk. AN SSSR No. 422, 1 (1967).

CERTAIN PROCESSES OF QUANTUM ELECTRODYNAMICS AT HIGH ENERGIES AND SMALL SCATTERING ANGLES

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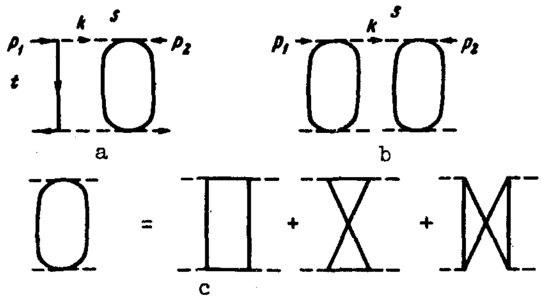
Submitted 2 September 1969
ZhETF Pis. Red. 10, No. 8, 399 - 402 (20 October 1969)

As is well known, the amplitudes of the γe and $\gamma\gamma$ scattering processes, calculated in the first nonvanishing order of perturbation theory at high energies $\sqrt{s}$ and at fixed momentum transfers $q = \sqrt{t}$, depends little on s, meaning that the cross sections of these processes decrease with increasing energy. In this paper we present results of calculations of the Feynman diagrams shown in the figure. The diagram corresponding to Delbruck scattering can be obtained by replacing the electron line in Fig. a by a line corresponding to a nucleus. In this order of perturbation theory, the elastic γe and $\gamma\gamma$ scattering do not decrease with increasing energy. Further iteration of the block shown in Fig. c leads to a series in terms of the quantity $\alpha^2 \ln s$ ($\alpha = 1/137$). The parameter $\alpha^2 \ln s$ becomes of the order of unity at fantastically high energies, but summation of this series is of interest from the general theoretical point of view of investigating the problem of the vacuum Regge pole in quantum electrodynamics.

The diagrams of the figure were calculated also by Cheng and Wu, whose results were recently published [1]. They actually considered not a photon but a neutral vector meson with mass λ. Their expression for the scattering amplitude at $t = 0$ in the limit as $\lambda \to 0$ contains an infrared divergence, which is not contained in the initial diagrams.

To calculate the asymptotic form of the diagrams, we used the Sudakov method [2], wherein the integration momenta are resolved into components $k_{\parallel}$, which lie in the plane p_1 and p_2, and $k_{\perp}$, which are perpendicular to this plane. In the integrals with respect to the perpendicular components of the photon momenta, which converge at the lower limit, it is



possible to omit the terms k^2 in the photon propagators $1/k^2$. This is precisely the situation in $\gamma\gamma$ scattering, where our results coincide fully with the results of Cheng and Wu. At the same time, for the case of γe scattering, the form of the amplitude depends strongly on the order of magnitude of the momentum transfer q. A characteristic quantity is the momentum $q_0 = 8/s$, for which the distance to the Karplus curve that is closest to the s-channel increases with increasing energy in the case of the diagram of Fig. a (the electron mass is that equal to unity throughout). In the region $q \sim 1 \gg q_0$, the amplitude A coincides with the expression of Cheng and Wu, in which it is necessary to put $\lambda = 0$, and is given by:

$$A = isI \tag{1}$$

Here I is the integral with respect to the Feynman parameters and to $k_\perp$, and is given in [1]. In the region $q \sim q_0$, the form of A is

$$\mathrm{Im}\,A = \frac{\alpha s}{2}\left\{ r_0^2\left(\frac{28}{9}\ln s - \frac{218}{27}\right)\delta_{\mu\nu} + \right.$$

$$\left. + \frac{1}{\pi}\int_4^\infty \frac{ds'}{s'}\left[\delta_{\mu\nu}\sigma(s')\left(1 - \frac{2+y^2}{2\sqrt{1+y^2}}L\right) + T_{\mu\nu}\tilde{\sigma}(s')\left(1 - \frac{y^2 L}{2\sqrt{1+y^2}}\right)\right]\right\}, \tag{2a}$$

$$\mathrm{Re}\,A = \frac{\alpha s}{2}\left\{\frac{14\pi r_0^2}{9}\delta_{\mu\nu} + \frac{1}{2}\int_4^\infty \frac{ds'}{s'}\left[\delta_{\mu\nu}\sigma(s')\left(y - \frac{y^2+2}{\sqrt{1+y^2}}\right) + \right.\right.$$

$$\left.\left. + T_{\mu\nu}\tilde{\sigma}(s')\left(y - \frac{y^2}{\sqrt{1+y^2}}\right)\right]\right\}. \tag{2b}$$

where

$$y = \frac{2s'}{\sqrt{-ts^2}}, \quad L = \ln\frac{\sqrt{1+y^2}+1}{\sqrt{1+y^2}-1}, \quad T_{\mu\nu} = \frac{q_\mu q_\nu}{|q|^2} - \frac{\delta_{\mu\nu}}{2},$$

$$\sigma(s') = 4\pi r_0^2 s'^{-3}\left[(s'^2 + 4s' - 8)\ln\frac{s' + \sqrt{s'(s'-4)}}{s' - \sqrt{s'(s'-4)}} - (s'+4)\sqrt{s'(s'-4)}\right],$$

$$\tilde{\sigma}(s') = 16\pi r_0^2 s'^{-3}\left[2\ln\frac{s' + \sqrt{s'(s'-4)}}{s' - \sqrt{s'(s'-4)}} - \sqrt{s'(s'-4)}\right],$$

$$\sigma(s') = \frac{\sigma_\perp + \sigma_\parallel}{2}, \quad \tilde{\sigma} = \sigma_\perp - \sigma_\parallel,$$

with $\sigma_\perp$ and $\sigma_\parallel$ having the physical meaning of the cross sections for the production of e^+e^- pairs upon scattering of two photons with mutually perpendicular or parallel polarization vectors, respectively. The indices μ and ν correspond to two possible polarizations of the incident and reflected γ quantum, and r_0 is the classical radius of the electron.

Thus, in the region $q \sim 1$ the amplitude is pure imaginary, whereas in the region $q \sim q_0$ the real and imaginary parts are comparable in magnitude: $\mathrm{Im}A \sim s \ln s$ and $\mathrm{Re}A \sim s$. The asymptotic form of the amplitude (2) corresponds, from the point of view of the j-plane, to the presence of an immobile "vacuum" pole at $j = 1$ and a family of "daughter" poles with residues that are singular at $t \to 0$. In the region $q_0 << q << 1$ both formulas (1) and (2) give an identical result

$$A = \frac{i\alpha s}{9} r_0^2 \left[\delta_{\mu\nu}\left(\frac{41}{3} - 7\ln(-t)\right) + 2T_{\mu\nu}\right]. \tag{3}$$

The amplitude corresponding to the Delbruck scattering can be obtained by making in (2) the substitution $s \to 2\omega$, where ω is the γ-quantum energy in the laboratory system. We then obtain a result different from the known formulas of Bethe and Rohrlich [3]. In the region $q << q_0$, expressions (2) give the previously known results [4, 5] for scattering by an atom without allowance for the screening of the nucleus. In the region $q \sim 1$, the differential cross section calculated with the aid of formula (1) must be multiplied by z^4 in the case of elastic γ-quantum scattering or by $(z^2 + z)^2$ in the case of scattering with disintegration of the atom, corresponding to the impulse approximation for scattering by the nucleus and by the electrons.

The light-light scattering amplitude corresponding to the diagram of Fig. b can likewise be represented in the form (1). It is easy to calculate with the aid of the optical theorem the cross section for the production of two pairs following gamma-quantum scattering

$$\sigma_T(2) = \frac{\alpha^2 r_0^2}{9\pi}\left[50\ln 2 - \frac{25\pi^2}{12} - \frac{19}{2}\right]. \tag{4}$$

We note that, unlike the cross section for the production of one pair, the cross section (4) does not decrease with increasing energy. These cross sections become comparable at energies $\sqrt{s}$ of the order of 3 GeV. This result may be of interest in astrophysics.

The authors thank V. N. Gribov and V. G. Gorshkov for useful discussions.

[1] H. Cheng and T. T. Wu, Phys. Rev. Lett. 22, 666 (1969).
[2] V. V. Sudakov, Zh. Eksp. Teor. Fiz. 30, 87 (1956); [Sov. Phys.-JETP 3, 65 (1956)].
[3] H. A. Bethe and F. Rohrlich, Phys. Rev. 86, 10 (1952).
[4] K. S. Suh and H. A. Bethe, Phys. Rev. 115, 672 (1959).
[5] F. Rohrlich and R. Gluckstern, Phys. Rev. 86, 1 (1952.

QUASINUCLEAR LEVELS IN THE NUCLEON-ANTINUCLEON SYSTEM

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Submitted 2 September 1969
ZhETF Pis. Red. 10, No. 8, 402 - 406 (20 October 1969)

We prove in this paper that S states can exist in a nucleon-antinucleon system with a small binding energy (compared with the nucleon mass). To this end, as in ordinary nuclear physics, we solve the nonrelativistic problem for bound states with a potential that describes the $N\bar{N}$ scattering correctly. An example of such a potential (corresponding to the exchange of scalar, pseudoscalar, and vector bosons) was given by Bryan and Phillips [1] (henceforth denoted the B-F potential), who obtained good agreement with the experimental up to 150 MeV in the c.m.s. However, the region of applicability of this potential apparently extends up to the region of about 300 MeV, where formation of the nearest resonant state of the nucleon (the Δ_{33} isobar) becomes possible.

The B-F potential contains tensor and spin-orbit forces. Estimates have shown that the role of the tensor interactions in the formation of the bound states of interest to us is small. The orbital momentum can therefore be regarded as a good quantum number in the first approximation, and the ground state is one of the S levels.

Since the B-F potential is certainly not the only potential capable of describing $N\bar{N}$ scattering at low energies, all the results obtained with its aid can be regarded as meaningful only if they are stable against variations of the form of the potential that leave the phase shifts unchanged. It is therefore convenient to solve the Schrodinger equation in an approximation that makes it possible to trace the dependence of the level position on the variations of the form of the potential. We chose for this purpose a stepwise approximation, wherein the true potential is replaced by several square wells. The parameters of the latter were chosen to satisfy the condition that the number of S levels (n) be the same in the true and in the approximate potentials. The number n satisfies the condition [2]

$$n \leqslant 2/\pi \int_0^\infty (-V)^{1/2}\, dr\,. \tag{1}$$

According to Levinson's theorem

$$n = \frac{1}{\pi}\ \delta(0)\,, \tag{2}$$

therefore the condition chosen by us limits the possible variations of the shape of the potential in such a way that the phase shifts remain practically constant at low energies.

To estimate the position of the levels, we considered only the potential for the ${}^1S_0^+$