at energies $\sqrt{s}$ of the order of 3 GeV. This result may be of interest in astrophysics.

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QUASINUCLEAR LEVELS IN THE NUCLEON-ANTINUCLEON SYSTEM

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We prove in this paper that S states can exist in a nucleon-antinucleon system with a small binding energy (compared with the nucleon mass). To this end, as in ordinary nuclear physics, we solve the nonrelativistic problem for bound states with a potential that describes the $N\bar{N}$ scattering correctly. An example of such a potential (corresponding to the exchange of scalar, pseudoscalar, and vector bosons) was given by Bryan and Phillips [1] (henceforth denoted the B-F potential), who obtained good agreement with the experimental up to 150 MeV in the c.m.s. However, the region of applicability of this potential apparently extends up to the region of about 300 MeV, where formation of the nearest resonant state of the nucleon (the Δ_{33} isobar) becomes possible.

The B-F potential contains tensor and spin-orbit forces. Estimates have shown that the role of the tensor interactions in the formation of the bound states of interest to us is small. The orbital momentum can therefore be regarded as a good quantum number in the first approximation, and the ground state is one of the S levels.

Since the B-F potential is certainly not the only potential capable of describing $N\bar{N}$ scattering at low energies, all the results obtained with its aid can be regarded as meaningful only if they are stable against variations of the form of the potential that leave the phase shifts unchanged. It is therefore convenient to solve the Schrodinger equation in an approximation that makes it possible to trace the dependence of the level position on the variations of the form of the potential. We chose for this purpose a stepwise approximation, wherein the true potential is replaced by several square wells. The parameters of the latter were chosen to satisfy the condition that the number of S levels (n) be the same in the true and in the approximate potentials. The number n satisfies the condition [2]

$$n \leqslant 2/\pi \int_0^\infty (-V)^{1/2}\, dr . \tag{1}$$

According to Levinson's theorem

$$n = \frac{1}{\pi}\ \delta(0) , \tag{2}$$

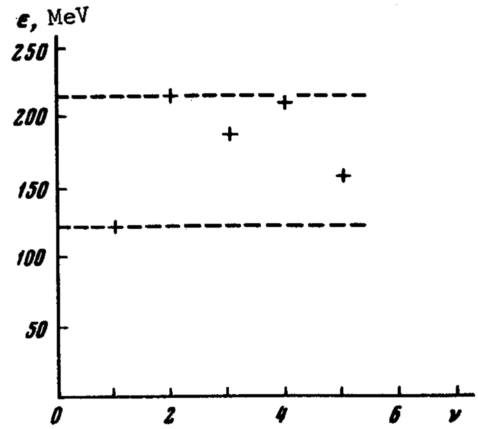
therefore the condition chosen by us limits the possible variations of the shape of the potential in such a way that the phase shifts remain practically constant at low energies.

To estimate the position of the levels, we considered only the potential for the $^1S_0^+$

state. It turned out that in this potential there is only one S level, whose energy (ε), with allowance for the uncertainty due to the variations of the form of the potential, lies in the range

$$120 \text{ MeV} \leqslant \varepsilon(^1S_0^+) \leqslant 220 \text{ MeV}. \quad (3)$$

This level corresponds to a meson resonance with quantum numbers $J^{PG} = 0^{-+}$ and a mass 1660 - 1730 MeV. The level shift resulting from the aforementioned variations of the potential form is shown in the figure.



Level positions vs. variations of the form of the potential. ν - number of "steps" approximating the true potential.

From the proximity of the potential curves for the states $^1S_0^+$, $^1S_0^-$, $^3S_1^-$, and $^3S_1^+$ we can expect the existence of meson resonances 0^{-+}, 0^{-+}, 1^{-+}, and 1^{--} with nearly equal masses.

The value of $\varepsilon(^1S_0^+)$ was obtained without allowance for the annihilation effects, which are described by the imaginary part of the B-F potential. The latter can change the binding energy by an amount of the order of several dozen MeV. Indeed, estimates of the simplest annihilation diagrams have shown that the annihilation radius should be smaller than or of the order of 0.1 F, whereas the distances that are significant in the present problem turn out to be larger than 0.5 F. We note that the influence of the potentials with small radius r_0 on the positions of the high-lying levels can be taken into account by perturbation theory in terms of the parameter $\sqrt{M\varepsilon}\, r_0$ [3], which in our case equals 1/5).

Having a slight effect on the position of the level, the annihilation processes fully determine the level width in this case. It was estimated with the aid of the formula

$$\Gamma = v\sigma |\Psi(0)|^2, \quad (4)$$

where σ is the cross section of the $p\bar{p}$ annihilation, v the relative c.m.s. velocity, and $\Psi(0)$ the value of the wave function of the system at zero. Using the experimental data on the $p\bar{p}$ annihilation ($v = 1.5 \times 10^{-15}$ cm^{3}sec^{-1} [4]), we obtained

$$\Gamma = 25 - 40 \text{ MeV}.$$

The error of this result, in view of the rough approximation made, amounts to 100%. Thus, the estimates gives grounds for concluding that the width of the predicted resonance can in any case not differ appreciably from the widths of the other known boson resonances.

It follows from our results that closely-lying resonances exist in the energy region 1580 - 1880 MeV; they have a width of the order of several dozen MeV and constitute quasi-nuclear states of the $N\bar{N}$ system.

There are experimental indications [5] of the existence of the predicted resonances. Although the quantum numbers of these resonances have not yet been established (even the total number of the resonances in this region is apparently still unknown), the experimental data seem to confirm the theoretical ideas concerning their nature.

In fact, even now, in this energy region of 300 MeV width, seven closely-lying resonances have already been observed, whereas in the energy region 0 - 1600 MeV the resonances are much more widely spaced [5]. In accordance with our prediction, the resonance widths range from 40 to 110 MeV, with an average distance on the order of 40 MeV between resonances (it must be emphasized that to interpret the observable spectra it is necessary to use the formulas of the theory of overlapping levels [6, 7]).

It is interesting to note that the peripheral interaction of an antinucleon with two nucleons should lead to the existence of a series of closely-lying baryon resonances (with baryon number 1) in the 2500 - 2800 MeV region. Similar states should occur also in many-nucleon systems with which an antinucleon is attached. It is important that a theoretical analysis of such resonances can be based on a quantum-mechanical formalism (such as the nonrelativistic many-body theory). Such many-nucleon resonances manifest themselves as excited states of nuclei, with an exitation energy on the order of 1.5 - 2 GeV.

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ISOTROPIC TURBULENCE IN A TURBULENT VISCOSITY FIELD

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It was shown in [1] that the principle of viscosity (molecular and turbulent) superposition makes it possible, if the parameters determining the turbulent viscosity are suitably chosen, to calculate the velocity distribution in the wall layer of tubes, as well as the law governing the resistance of tubes with smooth or rough walls.