The error of this result, in view of the rough approximation made, amounts to 100%. Thus, the estimates gives grounds for concluding that the width of the predicted resonance can in any case not differ appreciably from the widths of the other known boson resonances.

It follows from our results that closely-lying resonances exist in the energy region 1580 - 1880 MeV; they have a width of the order of several dozen MeV and constitute quasi-nuclear states of the $N\bar{N}$ system.

There are experimental indications [5] of the existence of the predicted resonances. Although the quantum numbers of these resonances have not yet been established (even the total number of the resonances in this region is apparently still unknown), the experimental data seem to confirm the theoretical ideas concerning their nature.

In fact, even now, in this energy region of 300 MeV width, seven closely-lying resonances have already been observed, whereas in the energy region 0 - 1600 MeV the resonances are much more widely spaced [5]. In accordance with our prediction, the resonance widths range from 40 to 110 MeV, with an average distance on the order of 40 MeV between resonances (it must be emphasized that to interpret the observable spectra it is necessary to use the formulas of the theory of overlapping levels [6, 7]).

It is interesting to note that the peripheral interaction of an antinucleon with two nucleons should lead to the existence of a series of closely-lying baryon resonances (with baryon number 1) in the 2500 - 2800 MeV region. Similar states should occur also in many-nucleon systems with which an antinucleon is attached. It is important that a theoretical analysis of such resonances can be based on a quantum-mechanical formalism (such as the nonrelativistic many-body theory). Such many-nucleon resonances manifest themselves as excited states of nuclei, with an exitation energy on the order of 1.5 - 2 GeV.

In conclusion, the authors consider it their pleasant duty to thank A. M. Badalyan, F. Calogero, and Yu. A. Simonov for useful discussions, and A. P. Sokolov for a discussion of the experimental situation.

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## ISOTROPIC TURBULENCE IN A TURBULENT VISCOSITY FIELD

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It was shown in [1] that the principle of viscosity (molecular and turbulent) superposition makes it possible, if the parameters determining the turbulent viscosity are suitably chosen, to calculate the velocity distribution in the wall layer of tubes, as well as the law governing the resistance of tubes with smooth or rough walls.

Turbulent (kinematic) viscosity is defined by the relation

$$V_T = kv\ell,$$

where v is the velocity scale and $\ell$ the length scale.

It follows from an analysis of the experimental data that the scale of the pulsational velocity in the layer next to the tube wall is the so-called dynamic velocity $v_* = \sqrt{\tau/\rho}$, where $\tau$ is the tangential stress and $\rho$ the density of the liquid. A feature of [1] is the fact that it considers not the constant value of the dynamic velocity, which is determined by the stress $\tau_0$ on the wall, but the velocity $v_*$ that depends on the stress $\tau$ at the given cross section, and is a scale of the pulsational velocity in the given cross section.

In this paper we apply the principle of superposition of the molecular and turbulent viscosities to an analysis of isotropic turbulence.

In this case the fundamental equation of the homogeneous isotropic turbulence can be written in the form

$$\frac{\partial B_d^d(r,t)}{\partial t} = \frac{1}{r^4}\frac{\partial}{\partial r} r^4(2\nu + 2\nu_T)\frac{\partial B_d^d(r,t)}{\partial r}, \tag{1}$$

where $B^d_d(r, t)$ is the second-order correlation function relating the longitudinal velocity pulsations,

$$B_d^d = \overline{u_d(x,t)\,u_d(x+r,t)}\,.$$

The superior bar denotes averaging, and $u_d$ is the velocity pulsation component in the direction defined by the points x and x + r.

Using third moments, we can write the well known Karman equation [2] for isotropic turbulence in the form:

$$\frac{\partial B_d^d}{\partial t} = \frac{1}{r^4}\frac{\partial}{\partial r} r^4\left(2\nu\frac{\partial B_d^d}{\partial r} + B_d^{dd}\right), \tag{2}$$

where

$$B_d^{dd}(r,t) = \overline{u_d(x,t)\,u_d^2(x+r,t)}$$

We choose for the scale of the turbulent viscosity the expression

$$\left[\int_0^r B_d^d(r,t)\,r\,dr\right]^{1/2}$$

or approximately

$$[B_d^d(0,t)]^{1/2} r\,.$$

Under this assumption we have the fundamental equation in the form

$$\frac{\partial B_d^d}{\partial t} = \frac{1}{r^4}\frac{\partial}{\partial r} r^4\left(2\nu + 2k\,[B_d^d(0,t)]^{1/2} r\right)\frac{\partial B_d^d}{\partial r}. \qquad (3)$$

Equations (2) and (3) give the connection between the second and third moments

$$B_d^{dd}(r,\ t) = 2k[B_d^d(0,t)]^{1/2} r\,\frac{\partial B_d^d(r,\ t)}{\partial r}\ . \qquad (4)$$

For the case of a negligibly small influence of the turbulent visconsity (K = 0) the solution of (3) takes the form

$$B_d^d(r,\ t) = \frac{e^{-r^2/8\nu t}}{4\,\nu t r^{3/2}}\int_0^\infty B_d^d(\xi,t)\,\xi^{5/2} I_{3/2}\left[\frac{\xi r}{4\nu t}\right] e^{-\xi^2/8\nu t}\,d\xi,$$

which yields under the condition

$$\int_0^\infty B_d^d(r,t)\,r^4 dr = \text{const} \qquad (5)$$

a self-similar solution of the point-source type

$$B_d^d(r,\ t) = t^n f(\eta) = C(4\nu t)^{-5/2}\exp(-r^2/8\nu t),$$

где $\eta = r t^{-1/2}$, $n = -5/2$, C is a constant [3].

L. G. Loitsanskii [4] has shown that if the second and third moments attenuate sufficiently rapidly at infinity, the condition (5) remains in force for Eq. (3). It is easy to see that (5) is valid for Eq. (3) if $r^5(\partial B_d^d/\partial r) = 0$ as $r \to \infty$. The quantity k which enters in (4) can depend in the general case on the time. We assume for K the expression

$$K = K_0(t/t_0)^m, \qquad (6)$$

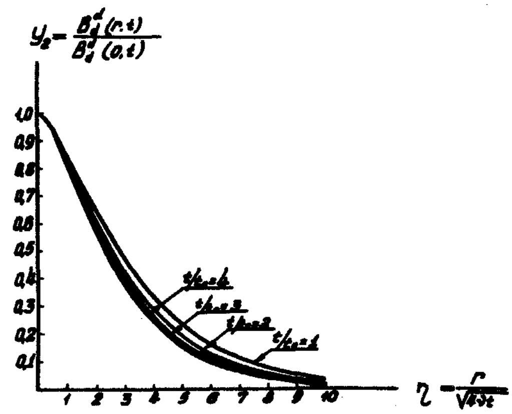
where $K_0$ and m are constants to be determined from the experimental data.

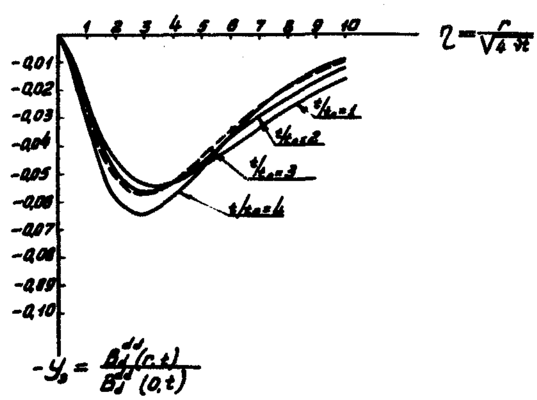
In the case when K is large and the molecular viscosity can be neglected, Eq. (3) also has a self-similar solution of the type

$$B_d^d = (t/t_0)^n f(\eta); \qquad \eta = (r/\ell_0)(t/t_0)^\beta,$$

where $\ell_0$ is the length scale corresponding to the instant of time $t_0$. We chose $\ell_0 = \sqrt{\nu t_0}$. From the self-similarity and invariance conditions it follows in this case that

$$\beta = -2(1+m)/7; \qquad n = -10(1+m)/7.$$





This solution takes the form

$$B_d^d(r, t) = \text{const}\left(\frac{t}{t_0}\right)^n \exp\left\{\left(\frac{r}{\ell_0}\right)\left(\frac{t}{t_0}\right)^\beta \frac{\beta\nu}{2K_0 \ell_0 v_0}\right\}. \tag{7}$$

In the particular case when m = 0, i.e., the coefficient K does not depend on the time, the exponents $\beta$ and n coincide with the well known Kolmogorov exponents [5]. We shall show, however, that the nonvanishing of the exponent m follows from the fact that, according to the experimental data, the ratio $B_d^{dd}(r, t)/[B_d^d(0, t)]^{3/2}$ varies in time like $(t/t_0)^m$, where $m = 1/9$.

In the general case, when it is necessary to take into account both the turbulent and the molecular viscosities, there is no self-similar solution. However, a good approximation at a self-similar solution, with some exponents $\beta$, can be obtained by considering the approximate equation obtained by averaging the time factor that appears in the term to which the given self-similar solution is foreign.

In the case of low viscosity, interest attaches to a "quasi-self-similar" solution close to a self-similar one with exponent $\beta = -2(1 + m)/7$. The factor $(t/t_0)^{1+2\beta}$ appearing in the case of molecular viscosity is replaced by its mean value $T_c$, and an approximate equation is obtained; its solution is given by

$$y_2 = \frac{B_d^d(r, t)}{B_d^d(0, t)} = e^{\beta\eta/2K_0}\left[1 + \frac{K_0}{T_c}\eta\right]^{-T_c\beta/2K_0^2}, \qquad (8)$$

whence

$$y_3 = \frac{B_d^{dd}(r, t)}{[B_d^d(0, t)]^{3/2}} = K_0 \frac{\nu}{v_0 \ell_0} \left(\frac{t}{t_0}\right)^m \frac{\eta^2}{1 + \eta K_0/T_c} y_2. \qquad (9)$$

The results of the calculations with $m = 1/9$, $k_0/\rho = -2.5$, and $v_0\ell_0/\nu = 14.7$ are shown in the figure. These data are in satisfactory agreement with the measurements of Stewart and Townsend [6], although they give a somewhat smaller deviation from self-similarity with respect to the variable $\eta/\sqrt{4\nu t}$ than would follow from the Stewart-Townsend measurements.

Further analysis is necessary to determine whether this deviation is the consequence of violation of the isotropy of the flow behind the grid, or whether it is necessary to use a more general solution of Eq. (4).

It should be noted that the calculated third moments agree better with the experimental data obtained by independent measurements than the second moments from which they have been calculated. It is therefore necessary to investigate experimentally the behavior of the second and third moments of isotropic turbulence.

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