FOURTH SOUND IN FERMI-BOSE QUANTUM LIQUIDS

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Much interest has been recently evinced in theoretical and experimental studies of Fermi-Bose quantum liquids. In practice these constitute either a superfluid solution of He^3 in He^4 at $T \ll T_{eq} = m^* v_F^2/2$, when the interaction between the Fermi excitations becomes appreciable, or pure He^3 in the temperature region where pairing can give rise to superfluidity.

In [1 - 4] they developed a theory of Fermi-Bose liquids and analyzed the acoustic solutions (first, second, and zero sounds) for the unbounded case. However, if the Fermi-Bose liquid fills sufficiently narrow channels, then the so-called fourth sound, i.e., a wave containing no oscillations of the normal component, can propagate in it, just as in pure He II [5] and in a nondegenerate superfluid He^3 - He^4 solution [6, 7]. The channel dimensions must satisfy in this case one of the following conditions: either the depth of penetration of the viscous wave or the mean free path of the elementary excitations must be appreciably larger than the transverse dimensions of the channels.

According to [2], the complete system of equations describing the acoustic oscillations of a Fermi-Bose liquid consists of the kinetic equation for the Fermi excitations, the continuity equation, and the equation of superfluid motion. The continuity equation for Fermi excitations and the equation of motion in a system moving with velocity $\vec{v}_s$ can be obtained from the kinetic equation by suitable integration. However, since it is assumed in the calculation of the fourth-sound velocity that the walls stop the motion of the normal component of the liquid in the entire volume [5, 6], we cannot use the momentum-conservation law. To calculate the fourth-sound velocity we should therefore derive from the kinetic equation only the continuity equation for the impurities, and introduce the following additional condition (which leaves the system of equations closed):

$$v_n = 0 \qquad \text{or} \qquad \mathbf{P}' = -\frac{m^* N}{1 + F_1/3} v_s .$$

The notation here and below is the same as in [2]: $\vec{P}'$ is the momentum of relative motion of the normal and superfluid components, m^* the effective mass of the Fermi excitation, N the density of the Fermi particles, and F_1 the first coefficient of the Legendre-polynomial expansion of the function $F(\chi)$ that describes the interaction of the excitations at $|\vec{p}| = |\vec{p}'| = p_F$; this interaction depends only on the angle χ between the vectors $\vec{p}$ and $\vec{p}'$.

Expressing $\vec{P}'$ in terms of the distribution function of the Fermi excitation, we arrive at a condition imposed on the variables contained in the equation:

$$\nu_1 = v_S/v_F(1 + F_1/3), \qquad (1)$$

where ν_1 is the first spherical harmonic of the function $\nu(\cos\theta)$, which is proportional to the deviation of the distribution function from the equilibrium value on the Fermi boundary (θ is the angle between the excitation momentum $\vec{p}$ and the wave vector $\vec{k}$).

Using the condition (1), we obtain ultimately the following system of linearized equations describing the propagation of fourth sound in a Fermi-Bose quantum liquid:

$$\nu_0 = 0$$

$$u\rho' - v_S\rho\left(1 - \frac{1}{1 + F_1/3}\,\frac{Nm^*}{\rho}\right) = 0\,, \qquad (2)$$

$$u\rho v_S - s^2\rho' = 0\,,$$

where ρ' is the deviation of the total density of the liquid from the equilibrium value in the sound wave, $u = \omega/k$, and $s^2 = \rho(\partial^2E/\partial\rho^2)$; E is the energy per unit volume of the liquid.

Vanishing of the zeroth spherical harmonic of the distribution function (first equation) corresponds to absence of oscillations of the normal component of the liquid. The condition for the compatibility of this system of equations yields a dispersion equation from which the fourth-sound velocity can be determined

$$u_4^2 = s^2\left(1 - \frac{1}{1 + F_1/3}\,\frac{Nm^*}{\rho}\right). \qquad (3)$$

Formula (3) is expressed in terms of variables that are natural for the case of pure He^3. This case is of physical interest in the sense that experimental observation of fourth sound in pure He^3 would be evidence of the onset of superfluidity (sound cannot propagate at all in pure He^3 filling narrow channels unless there is superfluidity).

For the case of a degenerate solution of He^3 in He^4 at low concentrations, we get

$$u_4^2 = v_1^2\left\{1 - \frac{Nm^*}{\rho_1(1 + F_1/3)}\left[\alpha_1(1 + F_1/3) + \frac{\delta m}{m^*}\right]^2\right\}, \qquad (4)$$

here u_1 is the velocity of first sound in the solution.

Separating the concentration dependence, we can obtain an expression relating the fourth-sound velocity in the solution with the fourth-sound velocity in pure He^4:

$$u_4^2 = u_{40}^2\left[1 + \left(\beta + \frac{m}{m^*}\right)\frac{Nm^*}{\rho_1}\right] \qquad (5)$$

α_1 and β are parameters connected with the dependence of the Fermi-excitation energy on the density ρ_1 of the Bose component [2], and

$$\frac{\delta m}{m^*} = 1 - \frac{m}{m^*}(1 + F_1/3).$$

Measurements of the first and fourth sound velocities in degenerate solutions make it possible to determine the effective masses of the impurity excitations and the parameters that characterize the interaction of the Fermi particles with either the Bose or the Fermi part of the liquid.

[1] I. M. Khalatnikov, ZhETF Pis. Red. 5, 288 (1967) [JETP Lett. 5, 235 (1967)].
[2] I. M. Khalatnikov, Zh. Eksp.Teor. Fiz. 55, 1919 (1968) [Sov. Phys.-JETP 28,1014 (1969)].
[3] J. Bardeen, G. Baym, and D. Pines, Phys. Rev. 156, 207 (1967).

[4] G. Baym, Phys. Rev. Lett. 18, 71 (1967).
[5] K. R. Atkins, Phys. Rev. 113, 962 (1959).
[6] D. G. Sanikidze and D. M. Chernikova, Zh. Eksp. Teor. Fiz. 46, 1123 (1964) [Sov. Phys.-JETP 19, 760 (1964)].
[7] B. N. Esel'son, N. E. Dyumin, E. Ya. Rudavskii, and I. A. Serbin, ibid. 51, 1064 (1966) [24, 711 (1967)].

MACROSCOPIC CAUSALITY-VIOLATION EFFECTS IN THE LEE AND WICK THEORY WITH INDEFINITE METRIC

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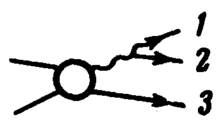
Recently Lee and Wick proposed a field-theory formulation that includes states with an indefinite metric (see [1]). The purpose of introducing an indefinite metric in [1] was, as usual, to the field-theory divergences. The authors of [1] propose and prove this possibility in a simple model, and also that no additional difficulties connected with violation of unitarity and causality arise in such a theory if it assumed that unusual states (with negative norm) are unstable with width $\Gamma \sim 10 - 50$ meV.

The unusual states (which we shall henceforth call LW particles) must appear as complex scattering-matrix poles lying on a physical sheet. We shall show that the existence of such poles at high energies should lead, in principle, to violation of causality on macroscopic scales.

Let the process



(1)

correspond to production of an LW particle and to its subsequent decay into two particles (1 and 2). For concreteness, we shall trace the behavior of particle 1.

In the rest system of the resonance, the coordinate of particle 1 is given by

$$R_0 = v_0(t - \tau_0), \quad (2)$$

where t_0 is the instant of registration, τ_0 is the lifetime of the unstable particle, and v_0 is the velocity of the particle 1. Unlike the normal Breit-Wigner resonance, in this case we have $\tau_0 < 0$, leading to the appearance of non-causal effects. In the Lee and Wick model, however, $|\tau_0| \sim \Gamma^{-1}\ 10^{-13}$ cm, so that no macroscopic causality violation takes place.

Let us consider now the decay of a rapidly moving resonance resulting from a collision at high energy. The coordinate of particle 1 can be obtained in this case with the aid of a Lorentz transformation of formula (2). It then turns out that

$$R = u\tau + v(t - \tau), \quad (3)$$

where R and t are the position of the first particle and the instant of its registration in the lab (t = 0 corresponds to the instant of collision in reaction (1) and R = 0 is the collision point), u and v are the velocities of the resonance and of particle 1, and $t = \tau/\sqrt{1 - u^2}$ is the lifetime of the resonance (LW particle) in the lab. A more rigorous proof of (3) is given in [2], which contains a space-time description of the reaction (1) with the aid of