$$n_{\rm eff} \sim p_0^3 \begin{cases} \exp(-\lambda/T), & \lambda_p = \lambda \\ (T/\lambda)^3, & \lambda_p = \lambda\cos\theta,\ \mathbf{E}_0 \parallel \mathbf{A} \end{cases}.$$

When the field E_0 is increased, "breakdown" occurs, wherein the electrons jump through the gap as a result of the energy acquired from E_0.

We note in conclusion that an experimental investigation of the effects under consideration is feasible at the present time. It calls for fields $E \approx 3 \times 10^4 - 10^5$ V/cm with $\lambda \sim 3 \times 10^{-3} - 10^{-2}$ eV.

The authors are grateful to V. M. Galitskii for useful discussions.

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GRAVITATIONAL RESONANT DETECTOR WITH TWO DEGREES OF FREEDOM

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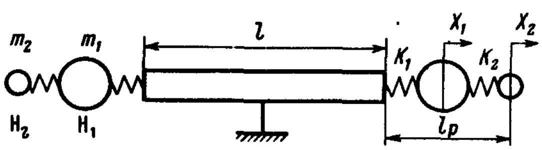
The sensitivity of resonant detectors for gravitational waves can be increased, as noted in [1], by increasing the initial displacement, this being accomplished by introducing a rigid (at the given frequency) rod in a break of the resonant coupling. In addition, one can use as a mechanical amplifier of resonant oscillations (as will be shown below) a system with two degrees of freedom. Such a detector is illustrated schematically in the figure.



Let us estimate first the permissible rod length in the field of a gravitational wave defined by a parameter h and a frequency ω, i.e., the length at which the condition $\Delta\ell \ll \zeta$ is still satisfied (here $\Delta\ell = F\ell/ES$ is the quasi-static displacement of the ends of the rod, and $\zeta = h\ell/2$ is the reduction of the spatial distance ℓ in the field of the wave). Rewriting the condition in the form $\zeta = n\Delta\ell$ ($n \gg 1$) and substituting in $\Delta\ell$ the expression for the force $F = \omega^2 m\ell h/4$ [2], we obtain for ℓ after simple transformations

$$\ell = \sqrt{\frac{2}{n}}\ \frac{v_s}{\omega}, \qquad (1)$$

where v_s is the velocity of propagation of the transverse oscillations in the rod.

Let us consider now a resonant system with two degrees of freedom (corresponding to one half of the detector, since the center of mass of the system is at rest in the field of the wave). The damping in the system is determined by the friction forces acting on the masses:

$$\begin{cases} m_1\ddot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 + H_1\dot{x}_1 = F_1 \\ m_2\ddot{x}_2 + k_2x_2 - k_2x_1 + H_2\dot{x}_2 = F_2 \end{cases}. \qquad (2)$$

We write down the system of equations in symmetrical form, introducing the notation $\alpha_{11} = k_1 + k_2$, $\alpha_{22} = k_2$, $\alpha_{12} = -k_2$, $\beta_{11} = m_1$, $\beta_{22} = m_2$, $\varepsilon_{11} = H_1$, and $\varepsilon_{22} = H_2$, with $\beta_{12} = 0$ and $\varepsilon_{12} = 0$:

$$\begin{cases} \beta_{11}\ddot{x}_1 + \alpha_{11}x_1 + \alpha_{12}x_2 + \epsilon_{11}\dot{x}_1 = f_1 e^{i\omega t} \\ \beta_{22}\ddot{x}_2 + \alpha_{12}x_1 + \alpha_{22}x_2 + \epsilon_{22}\dot{x}_2 = f_2 e^{i\omega t} . \end{cases} \tag{3}$$

We set the partial frequencies of the first and second resonators equal to each other:

$$n^2 = \alpha_{11}/\beta_{11} = \alpha_{22}/\beta_{22}.$$

We then obtain for the natural frequencies of the system, assuming H_1 and H_2 to be small quantities [3]:

$$\omega^2_{1,2} = n^2(1 \pm \gamma) \text{, where } \gamma^2 = \alpha^2_{12}/\alpha_{11}\alpha_{22} .$$

Let one of the natural frequencies coincide with the frequency of the driving force. We assume that the resonator Q factors are limited only by the observation time, $Q = \omega\tau/2$. Then

$$Q = \frac{m_1\omega}{H_1} = \frac{m_2\omega}{H_2} \text{, i.e. } H_1 = H_2(m_1/m_2) . \tag{4}$$

Since the displacement of the body in the field of the gravitational wave does not depend on its mass, we obtain a similar relation for the forces $f_1 = f_2m_1/m_2$. We seek a solution of the system (3) in the form $x_{1,2} = x_{1,2}e^{i\omega t}$. Using the general solution given in [4] for the system (3) and calculating x_1, x_2, and the determinant D of the system, we obtain

$$x_1 = \frac{f_1k_1}{D}\left(\frac{m_2}{m_1} + \frac{i}{Q} + \sqrt{\frac{m_2}{m_1}}\right), \quad x_2 = \frac{f_2k_2}{D}\frac{m_1}{m_2}\left(\sqrt{\frac{m_2}{m_1}} + \frac{i}{Q} + 1\right),$$

$$D = 2i\omega k_2 H_2\sqrt{m_1/m_2} .$$

Neglecting the first and the second terms in the parentheses in the expressions for x_1 and x_2 (with $m_1 >> m_2$) and making use subsequently of the smallness of m_2/m_1, we get:

$$x_1 = \frac{f_1}{2}\frac{Q}{k_1} \qquad x_2 = \frac{f_2}{2}\frac{Q}{k_2}\sqrt{\frac{m_1}{m_2}} .$$

Introducing the static displacements of the masses, $\delta_1^{st} = F_1/k_1$ and $\delta_2^{st} = 2F_2/k_2$, which can be easily obtained from (2) at x = const, we can write finally

$$x_1 = \frac{1}{2}\delta_1^{st}Q, \qquad x_2 = \frac{1}{4}\delta_2^{st}Q\sqrt{m_1/m_2} .$$

We thus obtain for the relative displacement ζ of small bodies m_2 in the complete detector system ($\delta^{st} = h(\ell + \ell_p)$):

$$\zeta = \frac{1}{2} h(\ell + \ell_p) Q \sqrt{m_1/m_2} \tag{5}$$

i.e., the displacements are larger by a factor $[m_1/m_2]^{1/2}$ than in a system with one degree of freedom. It must be noted that if the Q factors are not equal, then the worse Q factor enters in (5).

Let us determine now the fluctuation displacements of a small body. The fluctuation oscillations of the end of the rod, recalculated in terms of the displacements of a small body, are small compared with the fluctuation of the coordinate of the latter, due to the action of the residual gas on the resonant system. Using [5] the expression for the "Fourier components" of the coordinate fluctuations in the resonant regime $((x^2)_\omega)$, we write the fluctuation displacements in a frequency band Δf defined by the observation time, $\Delta f = 1/\tau$:

$$\overline{x_2^2} = (x^2)_\omega \Delta f = \frac{2kT\Delta f}{\omega^2 H_2} ,$$

where k is Boltzmann's constant and T the temperature.

We note that by virtue of condition (4) this estimate coincides with the estimate of the displacements of a small body, due to the fluctuation oscillations of the first body.

We present numerical estimates of the fluctuation displacements and the energy fluxes sufficient for the registration, taking as the radiator the pulsar in the Crab nebula with $T = 3.3 \times 10^{-2}$ sec ($\omega \approx 3.8 \times 10^2$ sec^{-1} and a gravitational-radiation power $P = 10^{38}$ erg/sec. corresponding to a flux $t \approx 10^{-5}$ erg/cm^{2}sec at the earth's surface.

At $\tau \approx 10^6$ sec and m = 100 g we get $x_{fluct} \approx 8 \times 10^{-12}$ cm.

Let us express the value of the flux in terms of the minimum observable displacements (ζ), using the formula given in [6] for the energy flux of the gravitational waves, $t = ch^2/2\kappa$ (where κ is Einstein's constant). To this end, we express h in terms of ζ from (5) and, substituting in the expression for t, we obtain:

$$t = \frac{2c\omega^2\zeta^2}{\kappa(\ell + \ell_p)^2 Q^2 \frac{m_1}{m_2}} .$$

Let us calculate first the value of ℓ. For n = 10, we obtain from (1) ℓ = 6.6 m. At a resonator-chamber length (superconducting suspension) $\ell_p \approx 1.5$ m [1], $Q = 2 \times 10^8$, $m_1/m_2 = 10^3$, and $\zeta = 10^{-11}$ cm, we obtain $t \approx 10^{-5}$ erg/cm^{2}sec.

It was assumed in this estimate of the flux that the signal will exceed the noise level. In this case, however, it is possible to use an accumulation method, which yields a gain in the signal/noise ratio by a factor $n = \tau/T$ (where τ is the observation time and T the period of the signal). A preliminary phase search or averaging over the phase decrease this ratio by 8 and 34% [7]. Thus, in the reception of a periodic signal, one can hope to register a flux $t \approx 10^{-10} - 10^{-11}$ erg/cm^{2}sec.

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THREE-PARTICLE PRODUCTION AMPLITUDE AT HIGH ENERGIES AND LARGE MOMENTUM TRANSFERS

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The asymptotic amplitude of elastic scattering $B(S, q^2)$ was investigated in [1, 2], using the theory of complex angular momenta, at high energies S (S is the square of the energy in the c.m.s.) and large momentum transfers q^2 (with $q^2 << S$ but $\alpha' q^2 \ln S/m^2 >> 1$, where α' is the slope of the Pomeranchuk trajectory at $q^2 = 0$). The problem was solved by summing the contributions, the so-called "Mandelstam branch points" in the angular-momentum plane, connected with the exchange of a certain number of Pomeranchuk poles. It was shown that at presently attainable energies the result does not depend too strongly on the detailed behavior of the jumps on the cuts in the angular-momentum plane. The simplest form is obtained by retaining in the contribution of the n-th branch point (n is the number of exchanged reggeons) only the factor $(-1)^n$ connected with the antiunitary character of the reggeon diagrams. Such an approximation is equivalent to neglecting the dependence of the reggeon-diagram vertices on the energy, the momentum trnasfer, and the number of emitted reggeons. The scattering amplitude then takes the form

$$B(\xi_1 q^2) \approx i s B_0 e^{-\sqrt{2\pi\alpha' q^2 \xi}} \cos(\sqrt{2\pi\alpha' q^2 \xi} + \chi_0), \quad \xi = \ln\frac{S}{m^2}. \tag{1}$$

Here χ_0 is the almost-constant phase, which depends little (logarithmically) on q^2 and ξ, and B_0 is a function containing no exponential dependence on q^2 and ξ.

We present in this paper results obtained for the asymtotic amplitude of three-particle production $a + b \to 1 + 2 + 3$ at high energies and large momentum transfers, using a method similar to that mentioned above. If we neglect, as in the other case, the dependence of the corresponding reggeon-diagram vertices on the energies, momentum transfers, and the number of emitted reggeons, then it can be readily shown that the production amplitude A is proportional to

$$A(\xi_{12}\xi_{23}, \vec{q}_1, \vec{q}_2) \sim B(\xi_{12}, q_1) B(\xi_{23}, q_2) + \int \frac{d^2k}{(2\pi)^2} B(\xi_1 k) B(\xi_{12}, q_1 k) B(\xi_{23}, q_2 k). \tag{2}$$

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