In conclusion, I am grateful to A.I. Tsygan and E. B. Gliner for useful discussions.

[1] G. Ya. Lavrent'ev, Zh. Tekh. Fiz. 39, 1316 (1969) [Sov. Phys.-Tech. Phys. 14, (1970)].
[2] J. Weber, General Relativity and Gravitational Waves, Interscience, 1961.
[3] S. P. Strelkov, Vvedenie v teoriyu kolebanii (Introduction to Oscillation Theory), Nauka, 1964.
[4] Kin N. Tong, Theory of Mechanical Vibration, Wiley, 1961.
[5] L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika (Statistical Physics), Nauka, 1964 [Addison-Wesley]
[6] L. D. Landau and E. M. Lifshitz, Teoriya Polya (Field Theory), Nauka, 1967 [Addison-Wesley]
[7] A. A. Kharkevich, Bor'ba s pomekhami (Combatting Noise), Nauka, 1965.

THREE-PARTICLE PRODUCTION AMPLITUDE AT HIGH ENERGIES AND LARGE MOMENTUM TRANSFERS

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Submitted 8 October 1969
ZhETF Pis. Red. 10, No. 10, 499 - 504 (20 November 1969)

The asymptotic amplitude of elastic scattering $B(S, q^2)$ was investigated in [1, 2], using the theory of complex angular momenta, at high energies S (S is the square of the energy in the c.m.s.) and large momentum transfers q^2 (with $q^2 \ll S$ but $\alpha' q^2 \ln S/m^2 \gg 1$, where α' is the slope of the Pomeranchuk trajectory at $q^2 = 0$). The problem was solved by summing the contributions, the so-called "Mandelstam branch points" in the angular-momentum plane, connected with the exchange of a certain number of Pomeranchuk poles. It was shown that at presently attainable energies the result does not depend too strongly on the detailed behavior of the jumps on the cuts in the angular-momentum plane. The simplest form is obtained by retaining in the contribution of the n-th branch point (n is the number of exchanged reggeons) only the factor $(-1)^n$ connected with the antiunitary character of the reggeon diagrams. Such an approximation is equivalent to neglecting the dependence of the reggeon-diagram vertices on the energy, the momentum trnasfer, and the number of emitted reggeons. The scattering amplitude then takes the form

$$B(\xi_1 q^2) \approx i s B_0 e^{-\sqrt{2\pi\alpha' q^2 \xi}} \cos(\sqrt{2\pi\alpha' q^2 \xi} + \chi_0), \quad \xi = \ln\frac{S}{m^2}. \tag{1}$$

Here χ_0 is the almost-constant phase, which depends little (logarithmically) on q^2 and ξ, and B_0 is a function containing no exponential dependence on q^2 and ξ.

We present in this paper results obtained for the asymtotic amplitude of three-particle production $a + b \to 1 + 2 + 3$ at high energies and large momentum transfers, using a method similar to that mentioned above. If we neglect, as in the other case, the dependence of the corresponding reggeon-diagram vertices on the energies, momentum transfers, and the number of emitted reggeons, then it can be readily shown that the production amplitude A is proportional to

$$A(\xi_{12}\xi_{23}, \vec{q}_1, \vec{q}_2) \sim B(\xi_{12}, q_1) B(\xi_{23}, q_2) + \int \frac{d^2k}{(2\pi)^2} B(\xi_1 k) B(\xi_{12}, q_1 k) B(\xi_{23}, q_2 k). \tag{2}$$

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Here $\vec{q}_1$ and $\vec{q}_2$ are the momenta transferred from particle a to 1 and from 3 to b, and are two-dimensional vectors lying in a plane perpendicular to the direction of the relative momentum of the incident particles; $\xi_{12} = \ln S_{12}$ and $\xi_{23} = \ln S_{23}$, where S_{12} and S_{23} are the squares of the corresponding pair energies. The logarithm of the total energy $\xi = \ln S$ is connected with ξ_{12} and ξ_{23}, namely, $\xi = \xi_{12} + \xi_{23}$.

We confine ourselves to the usual kinematic conditions: both pair energies are assumed to be large, but much smaller than the total energy (the so-called "truly inelastic" case).

If we substitute (1) in (2) and then evaluate the resultant integral by the saddle-point method (this method can be justified if $\alpha' q >> 1$, where q^2 and ξ are any of the quantities q_1^2, q_2^2, ξ_{12}, ξ_{23}), then we can obtain the following expression for the asymptotic amplitude:

$$A = A_0^{I} is\, e^{-\sqrt{2\pi\alpha'(q_1^2\xi_{12} + q_2^2\xi_{23} + 2q_1q_2\sqrt{\xi_{12}\xi_{23}}\sin\phi)}} \times$$

$$\times \cos\left(\sqrt{2\pi\alpha'(q_1^2\xi_{12} + q_2^2\xi_{23} + 2q_1q_2\sqrt{\xi_{12}\xi_{23}}\sin\phi)} + \chi_0\right),$$

$$\phi_1 < \frac{\pi}{2} + \arccos\sqrt{\frac{\xi_{12}}{\xi}},\ \phi_2 < \frac{\pi}{2} + \arccos\sqrt{\frac{\xi_{23}}{\xi}},\ \phi_1 + \phi_2 > \frac{\pi}{2} \qquad (3.1)$$

$$A = A_0^{II} is\, e^{-\sqrt{2\pi\alpha'}[\sqrt{\xi_{12}}q_1 + \sqrt{\xi_{23}}q_2]} \cos\left(\sqrt{2\pi\alpha'\xi_{12}}q_1 + \chi_0^{II}\right) \times$$

$$\times \cos\left(\sqrt{2\pi\alpha'\xi_{23}}q_2 + \tilde{\chi}_0^{II}\right), \qquad \phi_1 + \phi_2 < \frac{\pi}{2}, \qquad (3.2)$$

$$A = A_0^{III} is\, e^{-\sqrt{2\pi\alpha'}[\sqrt{\xi}q_1 + \sqrt{\xi_{23}}|q_1 - q_2|]} \cos\left(\sqrt{2\pi\alpha'\xi}q_1 + \chi_0^{III}\right) \times$$

$$\times \cos\left(\sqrt{2\pi\alpha'\xi_{23}}|q_1 - q_2| + \tilde{\chi}_0^{III}\right),\ \phi_1 > \frac{\pi}{2} + \arccos\sqrt{\xi_{12}/\xi}, \qquad (3.3)$$

$$A = A_0^{IV} is\, e^{-\sqrt{2\pi\alpha'}[\sqrt{\xi}q_2 + \sqrt{\xi_{12}}|q_1 - q_2|]} \cos\left(\sqrt{2\pi\alpha'\xi}q_2 + \chi_0^{IV}\right) \times$$

$$\times \cos\left(\sqrt{2\pi\alpha'\xi_{12}}|q_1 - q_2| + \tilde{\chi}_0^{IV}\right),\ \phi_2 > \frac{\pi}{2} + \arccos\sqrt{\xi_{23}/\xi} \qquad (3.4)$$

In formulas (3), $\phi_1(\phi_2)$ is the angle between the vector $\vec{q}_1$ ($\vec{q}_2$) and the vector $\vec{q} = \vec{q}_1 - \vec{q}_2$, $\phi = \pi - \phi_1 - \phi_2$ is the angle between $\vec{q}_1$ and $\vec{q}_2$. It is easy to verify that the four regions (3.1) - (3.4) cover all the possible values of ϕ_1 and ϕ_2. In the region (3.1), the value of the integral (2) is determined by a certain saddle-point value of $\vec{k}$, located inside the triangle made up of the vectors $\vec{q}_1$, $\vec{q}_2$, and $\vec{q}_3$. In the regions (3.2), (3.3) and (3.4), the main contribution to the integral is made by the points $\vec{k} = 0$, $\vec{k} = \vec{q}_1$, and $\vec{k} = \vec{q}_2$, respectively. The pre-exponential factors A_0^{I} ,..., A_0^{IV} and the almost-constant phases χ_0^{I}, ..., $\tilde{\chi}_0^{IV}$ are analogous to the corresponding quantities in (1).

The physical picture leading to the asymptotic form (1) for the elastic amplitude consists in the fact that the transfer of a large momentum is produced with the aid of an exchange of a large number of reggeons, and the momentum transferred in each of these exchanges is small, approximately $1/\sqrt{\xi}$. This is connected with the fact that reggeon exchange corresponds to a certain effective interaction that is not singular at small distances [2]. In the case of three-particle production, the essential difference is that in the regions (3.2), (3.3), and (3.4) it is more convenient for the particles (1, 3), (1, 2), and (2, 3) to echange only the minimal number of reggeons. In this case, say in the region (3.2), a large momentum is transferred by a large number of reggeons from particle 1 to particle 2, and then from 2 to 3, but not directly from 1 to 3 ($\vec{k} \approx 0$). Such a picture is apparently not connected with the details of the model under consideration and always takes place for the case of production.

Formulas (3.1) - (3.4) show that the cross section for the production of three particles should decrease with increasing momentum transfer, in analogy with elastic scattering, and should oscillate against the background of this decreases. However, whereas in region (3.1) there is only one oscillation frequency, there are already several (4 each) in the other regions). For example, in (3.2) we have $\omega_1 = 2\sqrt{2\pi\alpha'\xi_{12}}q_1$, $\omega_2 = 2\sqrt{2\pi\alpha'\xi_{23}}q_2$, and $\omega^{\pm} = |\omega_1 \pm \omega_2|$.

We note that the asymptotic amplitude of three-particle production at high energies and large momentum transfers was considered in [3] with the aid of the unitarity condition in the S channel. The formulas obtained there do not coincide with (3). Nor was this to be expected, however, since only two- and three-particle S-channel states were retained in [3], whereas the theory of complex angular momenta calls for inclusion of many-particle states in the direct channel (with the particle number of the order of the logarithm of the energy).

To describe elastic scattering at high energies and large momentum transfer, it is possible to develop a potential model [2 - 4], in which the role of the "potential" is played by exchange of one reggeon, and allowance for the multiple rescattering corresponds to the eikonal approximation [5]. A formula analogous to the eikonal formula can be derived also for the production amplitude. A particularly simple answer is obtained in the case when the particle production is due to exchange of the same poles ("potentials") that are responsible for the scattering, and in addition two of the three particles coincide in the final state with the initial ones (1 and 3 in our choice of kinematics).

This is precisely the situation occurring in the case considered above, that of exchange of Pomeranchuk poles. It is then possible to obtain the following formula for the production amplitude[1)]:

$$A = -\lambda v_{12}v_{23}\int e^{-iq_1\rho_{12} - iq_2\rho_{23}}\, e^{2i\delta_{13}(\rho_{13})}\left(e^{2i\delta_{12}(\rho_{12})} - 1\right) \times$$

$$\times \left(e^{2i\delta_{23}(\rho_{23})} - 1\right) d^2\rho_{12}d^2\rho_{23}, \quad \rho_{13} = \rho_{12} + \rho_{23}, \tag{4}$$

[1)] We use the following normalization of A: $A = \int \psi_f^* \hat{W} \psi_i d^3r_{12}d^3r_{23}$, where ψ_i and ψ_f are the wave functions of the initial and final states; $\hat{W}$ is the transition operator. Here $d\sigma/d\Gamma = 2\pi(\mu_{ab}/k_{ab})|A|^2$, where k_{ab} and μ_{ab} are the momentum and reduced mass of the particles in the initial state, and $d\Gamma = \delta(E_i - E_f)d^3\rho,\ \ d^3\rho_2/(2\pi)^6$.

where ρ_{12}, ρ_{23}, and ρ_{13} are the impact parameters, v_{12} and v_{23} the relative particle velocities, and the constant λ characterizes the vertex of the emission of the additional (2nd) particle.

The scattering phases δ_{ij} have the usual eikonal form [5]:

$$\delta_{ij}(\rho) = -\frac{1}{2v_{ij}}\int_{-\infty}^{+\infty} dz\, V_{ij}(\rho, z) \tag{5}$$

where V_{ij} is the interaction potential.

If we expand (4) in powers of the interaction V_{ij} then it can be shown, just as in the case of elastic scattering, that the individual terms of this expansion correspond to the contributions of the different branch points in angular-momentum plane, but a literal summation of these contributions in the form (4) neglects a number of relativistic effects.

A formula similar to (2) can be obtained by going over in (4) to the momentum representation.

A detailed article, containing a derivation of the results presented here, will be published in the Physical Review.

The authors are grateful to V. N. Gribov, I. T. Dyatlov, and E. Leader for useful discussions. A. A. Ansel'm wishes to express his gratitude to Eliott Leader for hospitality during his stay at Westfield College in London. G. A. Winbow is grateful to the British Science Council for awarding him a research fellowship.

[1] A. A. Ansel'm and I. T. Dyatlov, Yad. Fiz. 6, 591, 603 (1967) [Sov. J. Nuc. Phys. 6, 430, 439 (1968)].
[2] A. A. Ansel'm and I. T. Dyatlov, ibid. 9, 416 (1969) [9, 242 (1969)].
[3] V. V. Anisovich and O. A. Khrustalev, ibid. 9, 1258 (1969) [9, No. 6 (1969)].
[4] S. C. Frautschi and B. Margolis, Nuovo Cimento 56A, 1155 (1968)
[5] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics), Fizmatgiz, 1963, p. 566 [Addison-Wesley].

ERRATA

In the article by V. A. Karmanov and G. A. Lobov, Vol. 10, No. 7, p. 214, the first formula (line 11) should read

$$\mathcal{P} = -\frac{21\sqrt{7}}{40\sqrt{\pi}} - \beta\frac{\sqrt{\Gamma_{11}\Gamma_{01}}}{\Gamma_{11}+\Gamma_{01}}\cos(\delta_{11}-\delta_{01})\cos\theta$$

The second formula (line 14) should read

$$\mathcal{P} = -\frac{3}{2}\sqrt{\frac{15}{\pi}}\,\beta\frac{\sqrt{\Gamma_{21}\Gamma_{11}}}{\Gamma_{21}+\Gamma_{11}}\cos(\delta_{11}-\delta_{21})\cos\theta$$

In line 18, read:"... the level density of the excited nucleus is large," and not ",,, is small,".