and ceases to depend on the pump intensity (G is a constant introduced in [3]). A similar dependence of the line width on the pump intensity was obtained also for the m-th Stokes component of stimulated Raman scattering (Fig. 1).

2. The estimate (2) follows from the results of the solution of the following auxiliary problems: a) For the SMBS saturation regime, we determine (in the static approximation) the complex amplitude A and the correllation function $k(\tau)$ of the Mandel'shtam-Brillouin component (MBC) at $z = 0$ (beginning of the interaction region), assuming that at $z = \ell$ the MBC is a normal random process with amplitude A^{eq}, correlation function $k^{eq}(\tau)$, and width $\Delta\omega_{MB}^{eq}$. The value of A can be determined from the equation

$$\partial A/\partial z + \frac{1}{2} g I_L(z) A = 0$$

($I_L(z)$ is the spatial distribution of the excited radiation, determined by Tang's theory [2]) and from the boundary condition $A_{z=\ell} = A^{eq} = \rho e^{i\phi}$:

$$A \simeq e^{i\phi}\sqrt{I_L(0) - \langle I_0\rangle}, \quad I_0 e^{-g\ell I_0} \simeq \rho^2, \tag{3}$$

$\langle I_0\rangle = I_{thr}^{MB}$; the angle brackets denote statistical averaging. Assuming the relative fluctuations of the quantity I_0 to be small and that $g\ell I_0 \gg 1$ (this means that the scattered radiation is much weaker than the exciting radiation at $z \approx \ell$ as is usually the case), we get

$$\langle I_0\rangle e^{-g\ell\langle I_0\rangle} = \langle \rho^2\rangle \equiv I^{eq}, \quad I_{thr}^{MB} \simeq \frac{1}{g\ell}(\ln + \ln\ln)\frac{1}{g\ell I^{eq}}, \tag{4}$$

where I^{eq} is the equivalent MBC intensity at $z = \ell$.

According to (3), only the phase of the reflected light fluctuates in the saturation regime. Consequently, the sought relation between the correlation functions is given by ([4], p. 566)

$$k = \frac{\pi}{4}\,\frac{E(k^{eq}) - [1-(k^{eq})^2]k(k^{eq})}{k^{eq}} \sim k^{eq} + \frac{1}{8}(k^{eq})^2 + \dots,$$

where E and k are elliptic integrals. The obtained connection between $k(\tau)$ and $k^{eq}(\tau)$ differ little from linear. Thus, as a result of the nonlinear transformation $A^{eq} \to A$, the form of the MBC spectrum (and consequently also the line width) remains practically unchanged:

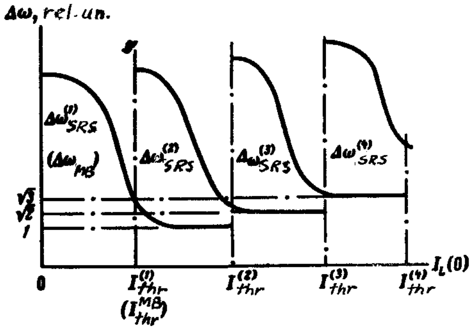

Fig. 1

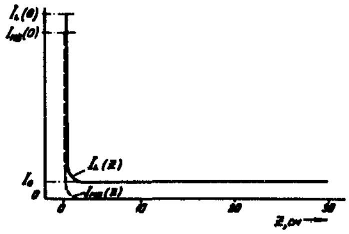

Fig. 2. Efficiency = $I_{MB}(0)/I_L = 0.9$, $g = 0.13$ cm/MW, $\ell = 30$ cm, $I_L(0) = 183$ MW/cm^2.

$$\Delta\omega_{MB} \approx \Delta\omega_{MB}^{eq}; \qquad (5)$$

b) We determine the line width $\Delta\omega_{MB}^{eq}$ for the MBC equivalent to real spatially-distributed sources upon saturation. Using the results of [2], it can be shown that in the greater part of the scattering region ($\ell_0 < z < \ell$, $\ell_0/\ell = 1/g\ell I_0 \ll 1$) the pump intensity $I_L(z)$ is constant and equal to I_0, while $I_{MB}(z) \ll I_0$. The intensities $I_L(z)$ and $I_{MB}(z)$ increase (by a factor 10 - 100) only in the relatively narrow interval $0 < z < \ell_0$, reaching approximately the same value at $z = 0$ (Fig. 2). Thus, the linear description of the SMBS is valid in the interval $\ell_0 < z < \ell$, and the amplification of the fields of the sources occurs at an almost constant pump level I_0, i.e., (see (1)), the SMBS line width gradually decreases in this interval

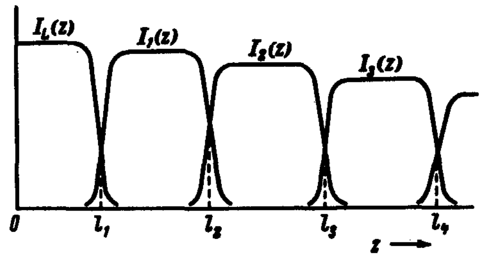

Fig. 3

$$\Delta\omega_{MB}(z) = \Delta\omega\sqrt{\frac{\ln 2}{g(\ell - z)I_0}}, \qquad l_0 < z. \qquad (6)$$

In the region $0 < \ell_0$ we have neither a further appreciable narrowing of the SMBS line (owing to the relatively small amplification of the MBC in this section), nor a noticeable broadening (see problem a). Thus, one can choose for $\Delta\omega_{MB}^{eq}$ the value of $\Delta\omega_{MB}(\ell_0)$ from (6), which leads to the estimate (2) when account is taken of relation (5), $\ell_0 \ll \ell$, and $I_0 \approx \langle I_0\rangle = I_{thr}^{MB}$.

3. Putting in (4) $I^{eq} = I_1/\sqrt{x}$ [2], we obtain the equation $xe^{-3/2x} = (g\ell I_1)^{2/3}$, from which we can estimate the threshold intensity:

$$x = g\ell\langle I_0\rangle = g\ell I_{thr}^{MB} = G \simeq \frac{3}{2}(\ln + \ln\ln)\frac{3}{2}(g\ell I_1)^{-2/3}.$$

Here

$$I_1 = \frac{\theta a n T f_L^2 k\sqrt{\ln 2}}{2\pi c} \simeq 6\cdot 10^{-34}\theta a n T f_L^2,$$

where θ is the solid angle of the laser-beam divergence, n and T are the refractive index and the temperature of the scattering medium, α the hypersound damping factor, f_L the pump frequency, k Boltzmann's constant, and c the velocity of light. For example, if g - 0.13 cm/MW (CS_2 [5]), $\theta = 10^{-4}$ sr, $\alpha = 300$ cm^{-1}, n = 1.5, T = 300°, $f_L = 4 \times 10^{14}$ Hz, and $\ell = 30$ cm, then $I_1 \approx 10^{-3}$ MW/cm^2, G = 8.5, and $I_{thr}^{MB} = 2.2$ MW/cm^3, and according to (2) the SMBS line is approximately one-third as narrow as $\Delta\omega$ upon saturation.

Stimulated Raman scattering. Upon successive excitation of the Stokes components of the SRS, the spatial distribution of the pump intensity $I_L(z)$ and of the Stokes components $I_m(z)$ (m = 1, 2, ...) takes the form shown qualitatively in Fig. 3 [6]. If we neglect damping, then the characteristic lengths ℓ_m and the saturation levels $(I_m)_{max}$ can be estimated as follows [7]:

$$\ell_m \simeq \frac{m\ln 1/a}{g_m I_L(0)}, \quad (I_m)_{max} = I_L(0)\frac{\omega_m}{\omega_L} \quad (m = 1,2,\ldots), \qquad (7)$$

where $g_m = g_0\omega_m/\omega_L$ is the gain for the m-th Stokes component with frequency ω_m, ω_L is the pump frequency, and $\alpha \approx 10^{-12}$ (for CS_2) is the initial relative level of the scattering intensity [8].

Using the same reasoning as in the case of SMBS, we can conclude that in the saturation interval of the m-th SRS component ($\ell_m < z < \ell_{m+1}$) only the phase of this component fluctuates, i.e., its complex amplitude as a randum function of the type of (5), namely $A_m \approx \sqrt{(I_m)_{max}}\ \exp(i\phi_m)$, with the spectrum of ϕ_m determined in the case of linear amplification in the preceding interval $\ell_{m-1} < z < \ell_m$ in the field of the (m - 1)st Stokes component with complex amplitude

$$A_{m-1} = \sqrt{(I_{m-1})_{max}}\ \ e^{i\phi_{m-1}},$$

etc. In the linear approximation, the linear approximation, the analysis of SMBS is similar to that of SRS [9], and it follows from the nonstationary SMBS theory developed in [10] that $\phi_m = \phi_{m-1} + \phi'_m$, where ϕ'_m are the phase fluctuations when $\phi_{m-1} = 0$, with ϕ_{m-1} and ϕ'_m statically independent. As a result, taking into account (7) and the Gaussian form of the scattering spectra in the linear region [2], we obtain the following estimate for the effective width of the spectrum of the m-th SRS Stokes component upon its saturation

$$\Delta\omega^{(m)}_{\text{SRS}} \approx \sqrt{\frac{m \ln 2}{\ln 1/a}}, \quad (m \approx 1, 2, \ldots) \tag{8}$$

i.e., the line width does not depend on the length of the scattering region, the pump intensity, or the gains, and is determined only by a and by the number m of the component (Fig. 1). The pump intensity thresholds indicated in Fig. 1 are $I^{(m)}_{\text{thr}} = (\ln 1/a)/g_m\ell$, where ℓ is the total length of the scattering region. If $\ell > \ell_m$ for all the significant pump intensity, then the estimate (8) remains in force also for an arbitrary distribution of the pump intensity over the beam cross section.

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N O T E

For technical reasons, the balance of the Russian Volume 10, No. 11 will be published in Volume 10, No. 12 of the translation.