INSTABILITY AND INTERMEDIATE STATE IN CURRENT-CARRYING CONDUCTORS

M. Ya. Azbel'
L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
Submitted 27 October 1969
ZhETF Pis. Red. 10, No. 11, 550 - 553 (5 December 1969)

1. We shall show that a sufficiently rapid change of the conductivity σ as a function of the magnetic field $\vec{H}$ can cause a conductor carrying direct current to become stratified into macroscopic or microscopic regions with different values of the conductivity (the analog of the intermediate or mixed state), and also the onset of a nonstationary mode. Such a change occurs, in particular, upon appearance of diamagnetic and periodic structures, metal-dielectric or dielectric-metal transitions with vanishing of the Landau bands (cf., e.g., [1]), in plates in a parallel field [2], etc.

In view of the complexity of the problem, we shall use in the rigorous formulation arguments based on simple and lucid examples.

2. Let $\vec{j} = \sigma(H)\vec{E}$ and $\vec{B} = \vec{H}$ ($\vec{j}$ - current density, $\vec{H}$ - magnetic field intensity), with $\sigma(H) = \sigma_0\theta(H/H_c)$, $\theta(x) = 1$ when $x \leq 1$, and $\theta(x) = 0$ when $x > 1$ ("metal-dielectric" transition). Then when a current $j = j(r)$ flows through a wire of radius R, we have initially, obviously, $H(r) = 2\pi\sigma_0 Er/c \equiv H_c r/r_c$. However, when $R > r > r_c$ there is no solution, since the "metallic" phase ($\sigma = \sigma_0$) leads to $H > H_c$ when $r > r_c$, i.e., to $\sigma(H) = 0$, and the "dielectric" phase ($\sigma = 0$) leads to $H < H_c$ and $\sigma(H) = \sigma_0$, so that a contradiction is obtained in both cases. (A similar contradiction can be easily obtained for $\sigma(H) = \alpha_1 + \beta_1 H + (\alpha_2 + \beta_2 H)\theta(H/H_c)$ at a definite ratio between the coefficients α_i and β_i). In this case it is impossible to obtain not only a symmetrical solution $H = H_\phi(r)$, but also any kind of stationary solution. Let us consider a more general case. Assume that when $H \leq H_c$ ("metal") the relation $\hat{\sigma} = \hat{\sigma}(H)$ is arbitrary, and when $H > H_c$ ("dielectric") we have $\hat{\sigma}(H) = 0$. Then $H \leq H_c$ everywhere in the conductor. In fact, $H \leq H_c$ on the metal-vacuum interface, and $H = H_c$ on the boundary with the dielectric. But outside the metal we have $H = \nabla\phi$ and $\nabla^2\phi = 0$, and the maximum moduli of both the harmonic function and of its gradient are reached on the boundary of the region. This means that it is impossible to have $H > H_c$ outside the metal, i.e., no dielectric phase can occur. Thus, the entire conductor is filled with metal in which $H \leq H_c$. Assume now, for simplicity, that the wire thickness $R \to \infty$. Then as $r \to \infty$ the problem becomes quasi-one-dimensional, $H' = 4\pi j(H)/c$. (Thus, when $\hat{\sigma}(H) = \sigma_0(H)\hat{\alpha}$, where $\hat{\alpha}$ is independent of H, j is parallel to the wire (z) axis, $H_z = 0$, and $r \to \infty$ yields $dH_\phi/dr = 4\pi j(H_\phi)/c$.) We see therefore that for a solution with $H \leq H_c$ to exist as $r \to \infty$ we need a divergence $\int^{H_c} dH/j(H) \sim r$, which calls for $\sigma'(H_c) \neq \infty$. (This is the necessary condition for the existence of an axially-symmetrical solution $H = H_\phi(r)$. A sufficient condition is that $\sigma'(H)$ be finite for all values of H.) On the other hand, if $\sigma'(H_c) = \infty$, there is no solution with $H \leq H_c$. According to the foregoing, this means that the problem has no symmetric solution at all, and that at a constant external potential difference on the ends of the wire there will appear in the wire either an

asymmetry of the field, or else a moving phase boundary $H = H_c$. (When $\sigma = \sigma_0\theta(H/H_c)$, only a nonstationary solution is possible.) The velocity v of the boundary motion is of the order of the Hall velocity, $v \sim cE/H_c$, from which we can obtain the oscillation frequency $\omega \sim v/R$, as well as the radiated power (see [3], Sec. 67).

3. Thus, the reason for the rigorous absence of a symmetrical solution is a singlularity of $\hat{\sigma}(H)$. An arbitrarily small change of $\hat{\sigma}(H)$, eliminating the infinite $\sigma'(H)$, leads to the appearance of a solution that tends asymptotically to H_c (see the formula for r(H) above). Such a solution, however, is absolutely unstable. This is best proved for large distances, where the one-dimensional case $H = H_y(x)$ takes place and the perturbations are plane waves in y and z. In the case when $\hat{\sigma}$ is diagonal and the perturbing additive is given by $E_{1z} = E_1(x) \exp(\lambda t)$ and $E_{1x} = E_{1y} = 0$, the equation for E_1 takes the form $E_1'' = f(x)E_1 + f'(d)E_1/f(d)$, where $f(x) = (4\pi/c^2)\sigma\{H_y(x)\}$, and $H_y(x)$ is the stationary solution. The substitution $E = \sqrt{f}\mathcal{E}$ leads to the well-studied (from the point of view of the asymptotic solution, the form of the solution for shallow and deep potential wells, etc) Schrödinger equation

$$\mathcal{E}'' = \left(\lambda f + \frac{3}{4} f'^2/f^2 - \frac{1}{2} f''/f\right)\mathcal{E}$$

with boundary conditions determined by the continuity of E and E' and the boundary with the vacuum. (In the case of an unbounded sample, it is necessary that E attenuate at infinity). In the case of small f", $f'' > f'^2/f$ at $x = x_0$ (ensuring a "potential well") there exists a solution at definite positive values of λ, i.e., instability sets in.

A similar analysis is always possible in the one-dimensional case (particularly when direct current flows in a wire of round cross section).

A physical cause of the instability of a direct current lies in the following: The current flowing through the sample produces in inhomogeneous magnetic field, causing an inhomogeneity of the conductivity and a self-consistent inhomogeneous distribution of the current density. Assume that in some region the current density experiences a fluctuation change, say an increase. This changes the magnetic field, increasing it in some places and decreasing in others. This in turn leads to a corresponding change of the conductivity. Since $\sigma'(H) \neq 0$, the "integral" conductivity can increase, leading to a growth in the flowing current, i.e., a growth of the fluctuation. Thus, an absolute instability of the initial current distribution sets in. This can result either in a distribution that does not have the initial symmetry of the problem, or in an oscillation corresponding to a nonstationary regime.

A stationary asymmetrical solution corresponds either to an intermediate case (if the surface energy is positive) or to a mixed state (for negative surface energy; the role of the correlation radius is played by the Larmor radius [1]).

We emphasize that in our analysis in Sec. 2 we used $\sigma(H)$ with a singularity only to simplify the analysis. As in clear from the foregoing, the instability of the stationary symmetrical current distribution is a rather general fact. It is caused by the essential nonlinearity of $\sigma(\vec{H})$ and nonlocality of the connection between $\vec{H}$ and $\vec{J}$.

A determination of the form of the asymmetrical and nonstationary solutions and of their stability against finite perturbations for arbitrary $\hat{\sigma}(\vec{H})$ and $\vec{B} = \vec{B}(\vec{H})$ ($\vec{B}$ - magnetic induction) is a highly complicated problem (which reduced to nonlinear Maxwell equations) and can be solved apparently only with a computer.

To observe these effects experimentally in the case of a large characteristic H, an appreciable fraction of this field can, of course, be obtained from an external source.

I am grateful to I. B. Levinson and E. I. Rashba for useful discussions.

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IONIZATION PROCESSES IN A LASER PLASMA

Yu.V.Afanas'ev, E. M. Belenov, O. N. Krokhin, and I. A. Poluektov
P. N. Lebedev Physics Institute, USSR Academy of Sciences
Submitted 27 October 1969
ZhETF Pis. Red. 10, No. 11, 553 - 557 (5 December 1969)

1. When powerful laser radiation interacts with matter, a multiply-ionized plasma is produced, and at relatively low radiation densities q the plasma can be regarded as being in a quasi-equilibrium state. Physically this is connected with the fact that the electron distribution function is cut off in this case at energies ε close to the threshold of the inelastic processes ($\varepsilon \sim I(z)$ is the ionization potential of an ion of multiplicity z), so that the ionization is brought about by the "tail" of the distribution function. At large q, however, one can expect an appreciable violation of the thermodynamic equilibrium in the plasma, owing to the fast electron diffusion in a region of energies greatly exceeding I(z). As a result, the effective electron temperature becomes larger than I(z), and the gap between the electron and ion temperatures (now limited mainly by the elastic electron-energy loss) amounts to $\Delta T \sim 4\pi Me^2 q/3m\omega^2 ck \approx 0.8 \times 10^{-16} q(M/m)$, where M is the ion mass and ω the frequency of the laser radiation. $\Delta T \lesssim 10^3$ eV when $q \approx 10^{12}$ W/cm^2. This occurs formally when

$$\beta_0 = \frac{I(z)\nu_i(z)}{\epsilon_0 \nu_{eff}(z)} < 1,$$

where ε_0 is the energy of electron oscillation in the field, and $\nu_i(z)$ and $\nu_{eff}(z)$ are the frequencies of the inelastic and elastic collisions [1]. Under the indicated conditions, the ionization state of the plasma differs greatly from the equilibrium state defined by the Saha formula.

The qualitative picture noted here has a direct bearing on the interpretation of the results of plasma diagnostics by means of bremsstrahlung or by observation of the ion emission lines. It is clear that in the region of fluxes corresponding to the condition $\beta_0 < 1$ the bremsstrahlung, say, will be harder than in the equilibrium case.

We consider below a model that makes it possible to calculate approximately, under strong-field conditions, such plasma parameters as the electron temperature, the ionization multiplicity, the radiation yield, etc.

2. The electron distribution function $F(\varepsilon, t)$ in a plasma, as a function of the energy