stability against finite perturbations for arbitrary $\hat{\sigma}(\vec{H})$ and $\vec{B} = \vec{B}(\vec{H})$ ($\vec{B}$ - magnetic induction) is a highly complicated problem (which reduced to nonlinear Maxwell equations) and can be solved apparently only with a computer.

To observe these effects experimentally in the case of a large characteristic H, an appreciable fraction of this field can, of course, be obtained from an external source.

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[1] M. Ya. Azbel', Usp. Fiz. Nauk 98, 601 (1969) [Sov.-Usp. 12, No. 4 (1970)].
[2] M. Ya. Azbel', Zh. Eksp. Teor. Fiz. 44, 1262 (1963) [Sov. Phys.-JETP 17, 85 (1963)].
[3] L. D. Landau and E. M. Lifshitz, Elektrodinamika sploshnykh sred (Electrodynamics of Continuous Bodies), Gostekhizdat 1957 [Addison-Wesley 1960].

IONIZATION PROCESSES IN A LASER PLASMA

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1. When powerful laser radiation interacts with matter, a multiply-ionized plasma is produced, and at relatively low radiation densities q the plasma can be regarded as being in a quasi-equilibrium state. Physically this is connected with the fact that the electron distribution function is cut off in this case at energies ε close to the threshold of the inelastic processes ($\varepsilon \sim I(z)$ is the ionization potential of an ion of multiplicity z), so that the ionization is brought about by the "tail" of the distribution function. At large q, however, one can expect an appreciable violation of the thermodynamic equilibrium in the plasma, owing to the fast electron diffusion in a region of energies greatly exceeding I(z). As a result, the effective electron temperature becomes larger than I(z), and the gap between the electron and ion temperatures (now limited mainly by the elastic electron-energy loss) amounts to $\Delta T \sim 4\pi M e^2 q/3m\omega^2 ck \approx 0.8 \times 10^{-16} q(M/m)$, where M is the ion mass and ω the frequency of the laser radiation. $\Delta T \lesssim 10^3$ eV when $q \approx 10^{12}$ W/cm^2. This occurs formally when

$$\beta_0 = \frac{I(z)\nu_i(z)}{\varepsilon_0 \nu_{eff}(z)} < 1,$$

where ε_0 is the energy of electron oscillation in the field, and $\nu_i(z)$ and $\nu_{eff}(z)$ are the frequencies of the inelastic and elastic collisions [1]. Under the indicated conditions, the ionization state of the plasma differs greatly from the equilibrium state defined by the Saha formula.

The qualitative picture noted here has a direct bearing on the interpretation of the results of plasma diagnostics by means of bremsstrahlung or by observation of the ion emission lines. It is clear that in the region of fluxes corresponding to the condition $\beta_0 < 1$ the bremsstrahlung, say, will be harder than in the equilibrium case.

We consider below a model that makes it possible to calculate approximately, under strong-field conditions, such plasma parameters as the electron temperature, the ionization multiplicity, the radiation yield, etc.

2. The electron distribution function $F(\varepsilon, t)$ in a plasma, as a function of the energy

ε at the instant of time t, satisfies the kinetic equation

$$\frac{\partial F}{\partial t} = \left(\frac{\partial F}{\partial t}\right)_q + \left(\frac{\partial F}{\partial t}\right)_{in} + \left(\frac{\partial F}{\partial t}\right)_{ee}. \quad (1)$$

The terms in the right side of (1) describe respectively the contributions made to the change of $F(\varepsilon, t)$ by the laser-radiation field, the inelastic electron-ion collision (including ionization), and the electron-electron collisions. We shall assume z to a continuous function of the time

$$z(t) = \frac{1}{N_0} \int_0^\infty F(\epsilon, t)\, d\epsilon + 1, \quad (2)$$

N_0 is the density of the neutral atoms (z - 2 corresponds to single ionization). The ionization potential is represented in the form

$$I(z) = \frac{I_H z^2}{n^2} \; [2],$$

n is the "effective" principal quantum number of the ionized shell and I_H is the ionization potential of the hydrogen atom. The frequency of the elastic collisions between the electrons and the ions of multiplicity z is given by

$$\nu_{eff}(z) = \frac{N_0 \sqrt{2\pi}\, e^4 (z-1)^2 \Lambda}{m^{1/2} \epsilon^{3/2}}, \quad (3)$$

where Λ is the Coulomb logarithm. For the inelastic-collision frequencies $\nu_i(z)$ at $\beta_0 < 1$ we can use the Born approximation, so that [2]

$$\nu_i(z) = \frac{\nu_{im}(1)}{z^3} \sqrt{\frac{I(z)}{\epsilon}}, \quad (4)$$

where $\nu_{im}(1)$ is the maximum frequency of inelastic collisions between the electrons and the neutral atoms. Under the indicated conditions, the expressions for $(\partial F/\partial t)_q$ and $(\partial F/\partial t)_{in}$ are [3]

$$\left(\frac{\partial F}{\partial t}\right)_q = -\frac{\epsilon_0}{3} \frac{\partial}{\partial \epsilon} \left\{ \nu_{eff} F - 2\nu_{eff} \epsilon \frac{\partial F}{\partial \epsilon} \right\},$$
$$\left(\frac{\partial F}{\partial t}\right)_{in} = -\frac{I(z)\nu_i(z)}{2\epsilon} \left\{ F - 2\epsilon \frac{\partial F}{\partial \epsilon} \right\}. \quad (5)$$

The collision term $(\partial E/\partial t)_{ee}$ can be represented in the Fokker-Planck approximation [4] in the form:

$$\left(\frac{\partial F}{\partial t}\right)_{ee} = (4\pi e^2)^2 \left(\frac{2\epsilon}{m}\right)^{1/2} \left\{ \frac{F^2}{8\pi\epsilon} - 2\epsilon \frac{\partial \psi}{\partial \epsilon} \frac{\partial}{\partial \epsilon}\left[\frac{\partial F}{\partial \epsilon} - \frac{F}{2\epsilon}\right] - \frac{\psi}{\epsilon}\left[\frac{\partial F}{\partial \epsilon} - \frac{F}{2\epsilon}\right] \right\},$$
$$\psi(\epsilon) = -\frac{1}{24\pi} \left\{ 3 \int_0^\epsilon F\, d\epsilon - \frac{1}{\epsilon} \int_0^\epsilon \epsilon F\, d\epsilon + 2\sqrt{\epsilon} \int_\epsilon^\infty \frac{F}{\sqrt{\epsilon}}\, d\epsilon \right\}. \quad (6)$$

We seek the distribution function $F(\varepsilon, t)$ in the form

$$F(\epsilon, t) = N_0 f(\epsilon, z) \exp\{\int_0^t \gamma(z) dt\}, \tag{7}$$

where $\int_0^\infty f(\varepsilon, z) d = 1$. In the case $z(t)$ satisfies the equation

$$\frac{dz}{dt} = \gamma(z) z, \quad z(0) = 2. \tag{8}$$

When $F(\varepsilon, t)$ of (7) is substituted in (1), we obtain in the left side of the equation $\gamma(z) \times (f + z(\partial t/\partial z))$. Since the function $f(\varepsilon, z)$ is large compared with $z(\partial f/\partial z)$ in the energy region corresponding to the maximum electron density, the term $z(\partial f/\partial z)$ can be neglected. In this case, Eq. (1) is satisfied, when account is taken of (5) and (6), if the function $f(\varepsilon, z)$ is close to Maxellian with an electron temperature $T(z)$ defined by the relation

$$T(z) = \frac{1}{2b}[(a^2 + 4b)^{1/2} - a], \tag{9}$$

where

$$a = A \frac{1}{\epsilon_0 (z-1)^2}; \; b = B \frac{1}{\epsilon_0 z^2 (z-1)^2}; \; A = \frac{3m^{1/2} I_{\mathrm{H}}^{3/2} \nu_{im}(t)}{\sqrt{2}\pi e^4 \Lambda n^3 N_0};$$

$$B = \frac{3\sqrt{3}\eta\, m^{1/2} I_{\mathrm{H}}^{1/2} \nu_{im}^{(1)}}{2\pi^{3/2} e^4 \Lambda n N_0}$$

η is the ratio of the maximum ionization frequency to the maximum frequency of all the inelastic processes when $z = 1$.

The cascade-development constant $\gamma(z)$ is connected with the electron temperature temperature as follows:

$$\gamma(z) = \frac{2\eta}{\pi^{1/2}} I_{\mathrm{H}}^{1/2} \nu_{im}(1) \frac{1}{n z^2 T^{1/2}}. \tag{10}$$

Formulas (7) - (10) solve the problem in principle. We present the explicit dependence of the temperature and of the ionization multiplicity on the time and on the flux density of the light field for $z > 2$:

$$T = T_0 \left(\frac{t}{\tau_0}\right)^{2/3} \sim q^{1/3} t^{2/3}, \quad z = \left(\frac{t}{\tau_0}\right)^{1/3} \sim q^{-1/12} t^{1/3}. \tag{11}$$

Here

$$T_0 \approx 3{,}8\sqrt{n\epsilon_0 I_{\mathrm{H}}}, \; \tau_0 = (\pi^{1/2} n / 6\eta \nu_{im}(1))(T_0 / I_{\mathrm{H}})^{1/2}.$$

For a radiation flux density $\sim 10^{13}$ W/cm^2, a pulse duration $\tau \approx 10^{-10}$ sec, $N_0 = 10^{20}$ cm^{-3} (in which case $\nu_{im}(1) \approx 5 \times 10^{12}$ sec^{-1}), and $\eta \approx 0.5$ we get from (11)

$$T \approx 10^2 n^{-1/3} I_{\mathrm{H}}, \quad z \approx 10\, n^{-5/12}. \tag{12}$$

It is easy to see that at the indicated pulse parameters we get $T \approx n^{5/2} I(z)$, so that when

$n \geq 2$ practically all the plasma electrons fall on the "Born tail" of the inelastic-collision cross sections. Formally this corresponds to satisfaction of the condition $\beta_0 < 1$ and to validity of the approximation (3) - (4).

In conclusion, we present numerical estimates for the ionization of a Ca plasma. In the case of the two outer shells of Ca (10 electrons) the mean value of n is of the order of 2.5 - 3. We consequently obtain from (12) $T \lesssim 80 I_H$ and $z \simeq 6 - 7$.

[1] Yu. V. Afanas'ev, E. M. Belenov, O. N. Krokhin, and I. A. Poluektov, Zh. Eksp. Teor. Fiz. 57, 581 (1969) [Sov. Phys.-JETP 30, No. 2 (1970)].
[2] I. A. Beigman and L. A. Vainshtein, X-radiation from the Solar Corona, FIAN Preprint No. 104, 1967.
[3] Yu. V. Afas'ev, E. M. Belenov, and O. N. Krokhin, Zh. Eksp. Teor. Fiz. 56, 256 (1969) [Sov. Phys.-JETP 29, 141 (1969)].
[4] B. A. Trubnikov, in: Voprosy teorii plazmy (Problems of Plasma Theory), Gosatomizdat, 1963.

RECONSTRUCTION OF Λ-N INTERACTION FROM THE EXACT SOLUTION OF THE FADDEEV EQUATIONS FOR HYPERTRITIUM

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Information on Λ-N interactions is obtained presently from Λ-N scattering at low energies and from the binding energies of light hypernuclei. However, experiments on Λ-p scattering (measurement of the total cross section) does not make it possible, because of the large experimental errors, to determine even the scattering length and the effective radius in the Λ-N interaction. The Λ-N interaction information extracted from the binding energies of the light hypernuclei depends strongly on the model assumptions that must be made in the calculations of many-particle systems.

The most complete analysis of the aggregate of experiments on elastic Λ-N scattering and the binding energies of the hypernuclei were made by Herndon and Tang [1]. Their analysis was based on the following assumptions: 1) the binding energy of the hypernuclei is calculated by a variational method, 2) in the determination of the Λ-N potential depths it is assumed that the motion of the Λ particle in the light hypernuclei can be represented as motion in a certain single-particle potential. The nucleon-nucleon potentials were chosen such as to be reproduce satisfactorily the parameters of the effective-radius theory in n-p scattering.

As the result of the analysis, it was concluded in [1] that for an optimal description of the Λ-N scattering and of the binding energies of the hypernuclei ${}_\Lambda H^3$, ${}_\Lambda H^4$, ${}_\Lambda He^4$, and ${}_\Lambda He^5$ it is necessary to introduce into the Λ-N potentials a repulsive core of ~ 0.6 F at an internal radius of about 2 F.

We present in this paper an analysis of the Λ-N interaction, based on a numerical so-

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