and

$$L_\mu^{(\pm)} = \frac{1}{2i}(K^{-1/2}\partial_\mu K^{1/2} \pm K^{1/2}\partial_\mu K^{-1/2}). \tag{7}$$

L' is the part of the Lagrangian which breaks the SU(3) × SU(3) symmetry and which we shall not specify here concretely.

In expression (5), f, α_F, and α_D are phenomenological coupling constants. The constant f determines the S-wave lengths for the scattering of mesons by baryons. The constants α_F and α_D correspond to the known constants of the F- and D-couplings of the baryon and meson octets.

[1] S. Weinberg, Phys. Rev. Lett. 18, 188 (1967).
[2] J. Schwinger, Phys. Lett. 24B, 473 (1967).
[3] J. Wess and B. Zumino, New York University Preprint, June, 1967.
[4] J. A. Cronin, Phys. Rev. 161, 1483 (1967).
[5] B. V. Struminskii, JINR Preprint R-2-3554, 1967.

SCATTERING OF AN ELECTRON NEAR THE FOCUS OF A LASER

S. P. Kapitza
Institute of Physics Problems, USSR Academy of Sciences

Submitted 15 March 1968
ZhETF Pis'ma 7, No. 10, 387-390 (20 May 1968).

From the elastic scattering of an electron by an electromagnetic field of light waves it is possible to determine the intensity of the field and its distribution near the focus of a laser beam.

The interaction of the electron with the electromagnetic field is in this case the result of second-order forces proportional to grad $\bar{E}^2$ and due to the time-averaged term $(e^2/2mc^2)|\bar{A}|^2$ in the Hamiltonian of the electron [1].

In the nonrelativistic case the interaction force $F_y = -\partial\varphi/\partial y$ can be readily expressed in terms of the average kinetic energy

$$\phi(x, y) = \frac{e^2\bar{E}^2(x, y)}{4m\omega^2},$$

which is possessed by the electron if it undergoes oscillations described by the equation

$$m\ddot{\xi} = eE(x, y)\sin\omega t$$

near the point (x, y) of an alternating field $E(x, y)\sin\omega t$ with a slowly varying amplitude. Essentially $\varphi(x, y)$ should be regarded as the potential of the field from which the electron is scattered. If the energy of the electron moving in the direction of the x axis is

$$eU = \frac{mv^2}{2} \gg \varphi$$

and the scattering is at a small angle α in the focal plane (x, y)

$$a = \frac{p_y}{p_x} = \frac{1}{p_x}\int_{-\infty}^{+\infty} F_y\,dt = \frac{1}{mv^2}\int_{-\infty}^{+\infty}\frac{\partial\phi}{\partial y}\,dx,$$

then we obtain in the impulse approximation the following expression for the angle of deflection of the electron as a function of the distance y:

$$a(y) = \frac{e^2}{4m^2\omega^2 v^2}\int_{-\infty}^{+\infty}\frac{\partial \bar{E}^2(x,y)}{\partial y}\,dx.$$

It is thus possible in principle to determine the distribution of the field intensity from the scattering picture.

Let us consider two cases of field distribution, formation of an imperfect focus and the case of ideal focusing.

The first case occurs in powerful lasers when the field in the focal region is a superposition of a number of independent wave systems. The effective intensity distribution I(r) is best described by a Gaussian distribution

$$I(r) = E^2 \exp\left(-\frac{r^2}{R^2}\right) 2\pi r\,dr = E^2 \exp\left(-\frac{x^2+y^2}{R^2}\right)$$

with an average focal-region radius R.

The power passing through the focal plane (x, y) is

$$P = \frac{c}{8\pi}\int_0^{\infty} E^2 \exp\left(-\frac{r^2}{R^2}\right) 2\pi r\,dr = \frac{cE^2R^2}{8},$$

and for the scattering angle we obtain the expression

$$a(y) = \frac{4e^2P}{m^2c^5\beta^2(KR)^2}\,\frac{y}{R}\exp\left(-\frac{y^2}{R^2}\right)\int_{-\infty}^{+\infty}\exp\left(-\frac{x^2}{R^2}\right)dx,$$

where $K = \omega/c = 2\pi/\lambda$ and $\beta = v/c$. We introduce further the characteristic power $P_0 = m^2c^5/e^2 = 8700$ MW for the interaction of strong electromagnetic waves with the electron [2], which is the fundamental physical power scale for the group of phenomena under consideration. We then obtain finally

$$a(\zeta) = \frac{4\sqrt{\pi}P}{P_0\beta^2(KR)^2}\,\zeta\exp\zeta^2,$$

where $\zeta = I/R$. The maximum angle is obtained when $\zeta = 1/\sqrt{2}$:

$$a_{max} = \frac{P\lambda^2}{\sqrt{2e\pi^3}\,P_0\,\beta^2R^2} = 2{,}26\,\frac{\lambda^2 P\ [\mathrm{MW}]}{R^2\,U\,[\mathrm{Volts}]}.$$

We note that when the frequencies $\omega_{1,2}$ of the different modes are close to each other we obtain a slowly-changing field structure and our consideration is valid only if the transit time τ through the inhomogeneity region satisfies the condition

$$\tau^{-1} = v/R > |\omega_1 - \omega_2|.$$

In the case of an ideal focus with one oscillation mode, the intensity is determined only by the power and by the diffraction angle of the spherical wave θ. In the case of diffraction from a square aperture, the approximate distribution of the field in the focal plane can be described by the expression

$$E(x, y) = E \frac{\sin K\theta y}{K\theta y} \frac{\sin K\theta x}{K\theta x}.$$

A calculation similar to that given above yields the following dependence of α on the parameter $\eta = K\theta y$

$$a(\eta) = \frac{2P\theta^2}{P_0\beta^2} \frac{\eta \sin 2\eta - 2\sin^2\eta}{\eta^3}; \ a_{max} = a(0{,}71\pi) = -\frac{0{,}62 P\theta^2}{P_0\beta^2}.$$

The scattering picture depends only on the power P, the aperture angle θ, and β^2, and does not depend on the wavelength of the light. We note that the field of intense electromagnetic waves can be regarded as a medium which has (in the case of nonrelativistic electrons) an effective refractive index

$$n(x, y) = \frac{\bar{v}(x, y)}{v} = \sqrt{1 - \frac{e^2\bar{E}^2(x, y)}{2m^2\omega^2 v^2}} \sim 1 - \frac{e^2\bar{E}^2(x, y)}{4m^2\omega^2 v^2} \quad \text{at } n \sim 1$$

connected with the energy density of the electromagnetic field [6]. When n = 0 and when the alternating-field intensity is

$$E = 2\omega\sqrt{\frac{Ue}{m}}$$

the electrons are completely reflected from the potential barriers (cf. [3]).

If on the other hand, when the atom is ionized, the electron is accelerated from the focal region to an energy determined by the potential φ.

The region of intense light waves can be investigated by electron microscopy, regarding the light as a transparent refracting medium. If a transmitted electron beam is used for the observation, the scattering can be increased by locating the focus near the virtual cathode of the electron-optical system, where the electron velocity is low and the interaction with the light is accordingly increased.

We note that electron scattering in a strong field of standing light waves was considered by P. L. Kapitza and Dirac [4] and was observed by Bartell [5]. In their case, however, the scattering is due to absorption and to stimulated emission of one or several quanta in the field of the intense plane light waves, and is proportional to the spectral energy density. In our case we deal with an interaction proportional only to the spatial energy density. The foregoing estimates can be generalized to include relativistic electrons as well as the case of standing and traveling waves in the region of the focus with more accurate account of their polarization. At powers $P > P_0/\theta^2$ and $\beta \sim 1$, the character of the phenomena changes somewhat and it is also necessary to take into account the magnetic interaction in the field of the focus. It is of interest to investigate self-focusing and acceleration of particles when such powers are propagated in a dense plasma.

After this paper was already submitted, N. E. Alekseevskii called my attention to a paper by Kibble [6], where a similar group of problems is considered.

In conclusion, I am grateful to Professor H. Messel for hospitality at the School of Physics of the Sydney University, where some of this work was performed, and to L. P. Pitaevskii for a discussion of the results.

[1] A. V. Gaponov and M. A. Miller, Zh. Eksp. Teor. Fiz. 34, 242 (1958) [Sov. Phys.-JETP 7, 168 (1958)]; F. B. Knox, Austral. J. of Physics 10, 221 (1957).
[2] S. P. Kapitza, Usp. Fiz. Nauk 88, 191 (1966) [Sov. Phys.-Usp. 9, 184 (1966)]; Zh. Eksp. Teor. Fiz. 41, 368 (1961) [Sov. Phys.-JETP 14, 266 (1962)].
[3] M. A. Miller, ibid. 35, 299 (1958) [8, 206 (1959)].
[4] P. Kapitza and P. Dirac, Proc. Cambridge Philos. Soc. 29, 297 (1933).
[5] Bartell et al., Phys. Rev. Lett. 14, 851 (1965).
[6] T. W. B. Kibble, Phys. Rev. 150, 1060 (1966).

LOWER BOUND OF ELASTIC SCATTERING AT A FIXED MOMENTUM TRANSFER

Nguyen Van Hieu
Joint Institute for Nuclear Research
Submitted 18 March 1968
ZhETF Pis'ma 7, No. 10, 391-393 (20 May 1968)

We establish in this paper a certain limitation on the decrease of the amplitude of elastic scattering $F(s, t)$ as $s \to \infty$ and for fixed $t < 0$, where s is the square of the total c.m.s. energy, $t = -2k^2(1 - \cos\theta)$, k is the c.m.s. particle momentum, and θ is the scattering angle in this system. For a number of processes, the amplitude $F(s, t)$ is an analytic function in t in the ellipse E_t (Martin's ellipse) with foci at the points $t = 0$ and $t = -4k^2$ and with major semi-axis $a = 2k^2 + \gamma$, $\gamma > 0$, as shown by Martin [1] and by Sommer [2]. The imaginary part of $F(s, t)$ will be denoted $A(s, t)$. From the results of Jin and Martin [3] it follows that

$$\max_{t \in E_t} |A(s, t)| \leqslant \text{const } s^{1+\epsilon}, \quad s \to \infty \tag{1}$$

for a certain positive $E < 1$. It is more convenient to consider in place of $A(s, t)$ the function