We note that electron scattering in a strong field of standing light waves was considered by P. L. Kapitza and Dirac [4] and was observed by Bartell [5]. In their case, however, the scattering is due to absorption and to stimulated emission of one or several quanta in the field of the intense plane light waves, and is proportional to the spectral energy density. In our case we deal with an interaction proportional only to the spatial energy density. The foregoing estimates can be generalized to include relativistic electrons as well as the case of standing and traveling waves in the region of the focus with more accurate account of their polarization. At powers $P > P_0/\theta^2$ and $\beta \sim 1$, the character of the phenomena changes somewhat and it is also necessary to take into account the magnetic interaction in the field of the focus. It is of interest to investigate self-focusing and acceleration of particles when such powers are propagated in a dense plasma.

After this paper was already submitted, N. E. Alekseevskii called my attention to a paper by Kibble [6], where a similar group of problems is considered.

In conclusion, I am grateful to Professor H. Messel for hospitality at the School of Physics of the Sydney University, where some of this work was performed, and to L. P. Pitaevskii for a discussion of the results.

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LOWER BOUND OF ELASTIC SCATTERING AT A FIXED MOMENTUM TRANSFER

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We establish in this paper a certain limitation on the decrease of the amplitude of elastic scattering $F(s, t)$ as $s \to \infty$ and for fixed $t < 0$, where s is the square of the total c.m.s. energy, $t = -2k^2(1 - \cos\theta)$, k is the c.m.s. particle momentum, and θ is the scattering angle in this system. For a number of processes, the amplitude $F(s, t)$ is an analytic function in t in the ellipse E_t (Martin's ellipse) with foci at the points $t = 0$ and $t = -4k^2$ and with major semi-axis $a = 2k^2 + \gamma$, $\gamma > 0$, as shown by Martin [1] and by Sommer [2]. The imaginary part of $F(s, t)$ will be denoted $A(s, t)$. From the results of Jin and Martin [3] it follows that

$$\max_{t \in E_t} |A(s, t)| \leqslant \text{const}\; s^{1+\epsilon}, \quad s \to \infty \tag{1}$$

for a certain positive $E < 1$. It is more convenient to consider in place of $A(s, t)$ the function

$$f(s,t) = \frac{A(s,t)}{A(s,0)}.$$

It has the same analytic properties with respect to t as $A(s, t)$. Let us assume that the total cross section behaves like $\text{const}\cdot s^{\rho}$ as $s \to \infty$. Then $A(s, 0) \sim \text{const}\cdot s^{1+\rho}$, $s \to \infty$, and we have

$$\max_{t \in E_t} |f(s,t)| \leqslant \text{const } s^{\epsilon - \rho}, \quad s \to \infty. \tag{3}$$

By making a change of variables $w = t + 2k^2$, we transform E_t into an ellipse E_w with foci at the points $w = \pm c$, where $c = 2k^2$, and with the same major semi-axis a. The minor semi-axis is $b = \sqrt{a^2 - c^2}$. We now introduce an arbitrary positive fixed number $\alpha < c$ and consider the ellipse E_w' with foci at the points $w = \pm c'$, $c' = c - \alpha$, and with minor semi-axis b. This ellipse, of course, lies inside E_w. The major semi-axis a' of the ellipse E_w' is given by

$$a'^2 = a^2 + c'^2 - c^2.$$

We choose c' (i.e., α) such that the points $w = \pm c$ (i.e., $t = 0$ and $t = -4k^2$) lie inside E_w', i.e., such that the physical region lies entirely in E_w'. This condition is satisfied if $a' > c'$, i.e., if

$$(2k^2 + \gamma)^2 + (2k^2 - \alpha)^2 - (2k^2)^2 > (2k^2)^2.$$

For large s we get from this inequality

$$\alpha < \gamma. \tag{4}$$

Using the conformal mapping

$$\xi = \frac{w + \sqrt{w^2 - c'^2}}{c'}$$

we transform the segment $[-c', c']$ in the w plane into a unit circle in the ξ plane, following Cerulus and Martin [4]. In this mapping the ellipse is transformed into a ring with small radius I and large radius R, where

$$R = \frac{a' + \sqrt{a'^2 - c'^2}}{c'}. \tag{5}$$

The point $w = c$ (i.e., $t = 0$) is transformed into the point $\xi = r$,

$$r = \frac{c + \sqrt{c^2 - c'^2}}{c'}. \tag{6}$$

We denote by m the maximum value of $|f(s, t)|$ in the interval $-c \leq w \leq c'$, i.e, in the interval $-4k^2 + \alpha \leq t \leq \alpha$, and by M the maximum value of $|f(s, t)|$ on the boundary of the ellipse E_t. According to Hadamard's three-circle theorem (see [5], theorem 5.32), we have

$$\ln|f(\Delta, 0)| \leqslant \left(1 - \frac{\ln r}{\ln R}\right)\ln m + \frac{\ln r}{\ln R}\ln M.$$

Substituting in (5) and (6) the values of a', c, and c', and then letting s go to infinity, we can readily see that

$$\frac{\ln r}{\ln R} \sim \sqrt{\frac{a}{\gamma}}.$$

On the other hand, $f(s, 0) = 1$. We therefore have

$$m \geqslant \left(\frac{1}{M}\right)^{\sqrt{a/\gamma}\,(1-\sqrt{a/\gamma})}.$$

For πN scattering we have $\gamma = 4m_\pi^2$. Using the condition (3) we now obtain

$$\max_{-4k^2+a \leqslant t \leqslant -a} |f(s, t)| \geqslant \text{const}\ s^{-(\epsilon'-\rho)\phi(a)}, \tag{7}$$

where

$$\phi(a) = \frac{\sqrt{a/4m_\pi^2}}{1 - \sqrt{a/4m_\pi^2}}. \tag{8}$$

If we assume that $A(s, t)$ is analytic in the entire t plane with cut on the real axis and that $A(s, t)$ is uniformly and polynomially bounded in the entire t plane as $s \to \infty$, then we get

$$\max_{-4k^2+a \leqslant t \leqslant -a} |f(s, t)| \geqslant \text{const}\ s^{-n\psi(a)}, \tag{9}$$

where

$$\Psi(a) = \frac{[1 - (1 + a/4m_a^2)^{-1/2}]^{1/2}}{1 - [1 - (1 + a/4m_\pi^2)^{-1/2}]^{1/2}}, \tag{10}$$

and n is a constant such that $|f(s, t)| \leq \text{const.}s^n$, $s \to \infty$ for all t.

If the scattering amplitudes have a Regge behavior, then the inequalities (7) and (9) constitute lower bounds for the corresponding Regge trajectories.

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On p. 309, line 15, read "$a' > c$ ..." in lieu of "$a' > c'$ ..."

On p. 310, line 2, read "$\ln|f(s, 0)|$..." in lieu of "$\ln|f(\Delta, 0)|$..."

line 7, read "$m \geq (1/M)\sqrt{\alpha/\gamma}/(1 - \sqrt{\alpha/\gamma})$" in lieu of "$m \geq (1/M)\sqrt{\alpha/\gamma}(1 - \sqrt{\alpha/\gamma})$"