tions, which assume the following form:

$$\text{curl}\, e = -\frac{1}{c}\frac{\partial \sigma}{\partial t}, \tag{1}$$

$$\text{curl}\,[b - 4\pi(b\nabla)_{B=B_0} M(B)] = \frac{4\pi}{c}\hat{\sigma} e, \tag{2}$$

where $\vec{e}$ and $\vec{b}$ are the vectors of the electric field and magnetic induction of the wave, $\vec{B} = \vec{B}_0 + \vec{b}$, $\vec{B}_0$ is the constant field in the sample, and $\hat{\sigma}$ is the conductivity tensor.

The dispersion relation obtained from (1) and (2) for a spherical Fermi surface is

$$k^2 = \frac{4\pi Ne\omega}{c\sqrt{q}H\cos\theta}, \tag{3}$$

$$q = 1 - 4\pi\sin^2\theta\frac{\partial M}{\partial B}, \tag{4}$$

where θ is the angle between $\vec{k}$ and $\vec{B}_0$.

Since the vector $\vec{b}$ of the wave is always parallel to the surface of the metal, the second term in the left side of (2), which is responsible for the observed oscillations, is equal to zero for a spherical Fermi surface if the field $\vec{B}_0$ is directed along the normal. If part of the Fermi surface has an elongated nearly cylindrical form, then the contribution to the oscillations made by this part is zero if the axis of the cylinder is directed along the normal. These conclusions agree with the experiments of [2,3].

Thus, on the basis of the presented experimental results and their interpretation, we can state that the oscillations observed in the propagation of helicons in metals are in the local limit a manifestation of the de Haas - van Alphen effect. The relative amplitude of the oscillations of the resonant frequency of the sample (Fig. b) makes it possible, as follows from (3) and (4), to measure directly the amplitude of the oscillations of $\partial\vec{M}/\partial\vec{B}$.

In conclusion, the authors are indebted to V. F. Gantmakher for a discussion of the results, to I. P. Krylov for useful discussions and for supplying the still-unpublished experimental data, and to S. F. Kosterev for technical help.

[1] C. C. Grimes, Plasma Effects in Solids, Dunod, 1965, p. 87.
[2] I. P. Krylov, Zh. Eksp. Teor. Fiz. 54, 1738 (1968) [Sov. Phys.-JETP 27, in press].
[3] P. B. Miller and P. C. Kwok, Phys. Rev. 161, 629 (1967).
[4] Y. R. Houck and R. Bowers, RSI 35, 1170 (1964)

STIMULATED SCATTERING OF LIGHT BY A LIQUID SURFACE

F. V. Bunkin, A. S. Samokhin, and M. V. Fedorov
P. N. Lebedev Physics Institute, USSR Academy of Sciences
Submitted 19 March 1968; resubmitted 9 April 1968
ZhETF Pis'ma 7, No. 11, 431-433 (5 June 1968)

In spontaneous scattering of low-intensity light from thermal fluctuations of a liquid surface there is no reaction of the electromagnetic field on the interface [1-3]. However, if the intensity of the light is sufficiently large, such a reaction, which is nonlinear in the field, becomes appreciable and can lead to a buildup of capillary waves, i.e., to stimulated scattering of light by the surface of the liquid. This effect is due to the ponderomotive action of the electromagnetic field and to the presence of an interface, and the liquid can be regarded as incompressible, just as in the case of free capillary waves.

Let us consider the incidence of a plane electromagnetic wave $\vec{E}_0(\vec{r}, t) = \vec{E}_0\exp[i(k_x x + k_z z - \omega t)]$, the polarization of which is perpendicular to the incidence plane (x, z), on the surface of a liquid having a dielectric constant $\epsilon > 1$; it is assumed that $\epsilon_0 = 1$ in the free space over the liquid and that the magnetic permeability is $\mu = 1$ throughout. The boundary conditions for the field on the liquid surface $z = \zeta$, without allowance for relativistic corrections, are [2]

$$[\mathbf{n}, \tilde{\mathbf{E}} - \mathbf{E}] = 0; \ \mathrm{curl}(\tilde{\mathbf{E}} - \mathbf{E}) = 0, \tag{1}$$

where $\vec{n}$ is the normal to the surface of the liquid. The field over the field is $\vec{\tilde{E}} = \vec{E}_0(\vec{r}, t) + \vec{E}_1 + \vec{E}^{(1)}$ and in the liquid $\vec{E} = \vec{E}_2 + \vec{E}^{(2)}$, where the fields $\vec{E}_1$ and $\vec{E}_2$ are determined by the Fresnel formulas for a flat surface $z = 0$, and the fields $\vec{E}^{(1)}$ and $\vec{E}^{(2)}$ are due to the deviation of the surface from a plane. We shall henceforth confine ourselves to capillary waves of the form $\zeta = \zeta_q\exp[i(qx - \Omega t)]$, $k|\zeta_q| \ll 1$, $q|\zeta_q| \ll 1$, where $2\pi/k$ is the wavelength of the incident light. In this case the propagation directions of the waves $\vec{E}^{(1)}$ and $\vec{E}^{(2)}$ lie in the incidence plane, and each of them contains components with frequencies $\omega \pm \mathrm{Re}\,\Omega$ and with wave vectors $\{k_x \pm q, 0, \sqrt{k^2 - (k_x \pm q)^2}\}$ and $\{k_x \pm q, 0, -\sqrt{\epsilon k^2 - (k_x \pm q)^2}\}$ in the liquid. The amplitudes of these waves, as in the case of spontaneous scattering [1,2] can be expressed linearly in terms of ζ_q.

When account is taken of the ponderomotive action of the electromagnetic field, the equations of the hydrodynamics of a viscous incompressible liquid assume the form

$$\rho\frac{\partial \mathbf{v}}{\partial t} = \eta\,\Delta\mathbf{V} - \nabla p + \rho\frac{\partial\epsilon}{\partial\rho}\nabla\frac{E^2}{8\pi}, \ \mathrm{div}\,\mathbf{V} = 0. \tag{2}$$

with boundary conditions on the surface

$$\begin{gathered} p + \left(\epsilon - 1 - \rho\frac{\partial\epsilon}{\partial\rho}\right)\frac{E^2}{8\pi} - 2\eta\frac{\partial v_z}{\partial z} + \alpha\frac{\partial^2\zeta}{\partial x^2} = 0, \\ \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} = 0, \end{gathered} \tag{3}$$

where $\vec{v}$, ρ, p, η, and α are respectively the velocity, density, pressure, viscosity, and surface-tension coefficient. Introducing in (2) and (3) the new quantity

$$p' = p - \rho\frac{\partial\epsilon}{\partial\rho}\frac{E^2}{8\pi},$$

we arrive at the ordinary problem of capillary waves (see, e.g., [4]), but with modified boundary conditions

$$p' + (\epsilon - 1)\frac{E^2}{8\pi} - 2\eta\frac{\partial v_z}{\partial z} + a\frac{\partial^2 \zeta}{\partial x^2} = 0,$$
$$\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} = 0, \tag{4}$$

where it is necessary to retain in the expression for E^2 only the terms with the frequencies of the capillary waves. In the weak-damping approximation we get the following solution of the characteristic equation:

$$\Omega = \pm\Omega_0 - 2iq^2\frac{\eta}{\rho} \pm iD\frac{q^2E_0^2}{\rho\Omega_0}, \quad \Omega_0 = \left(\frac{aq^3}{\rho}\right)^{1/2},$$
$$D = \frac{\epsilon - 1}{8\pi}\frac{k_z^2}{q}\frac{\sqrt{k^2-(k_x+q)^2} - \sqrt{\epsilon k^2-(k_x+q)^2} - \sqrt{k^2-(k_x-q)^2} + \sqrt{\epsilon k^2-(k_x-q)^2}}{(|k_z| + \sqrt{\epsilon k^2 - k_x^2})^2}, \tag{5}$$

which differs from the ordinary solution for capillary waves (see [4]) in the presence of the last term.

If $|D|E_0^2 > 2\Omega_0\eta$, then the amplitude of the capillary waves can build up in time in accordance with this condition, wherein one of the two traveling capillary waves with specified value of the wave vector q will be amplified. For a specified intensity I of the incident light, the same inequality $I > c\eta\Omega_0/4\pi|D|$ determines also the range of angles in which the stimulated scattering of the light by the capillary waves can be observed.

It is possible to obtain in similar fashion the conditions for stimulated scattering in the case when the wave vector of the scattered wave does not lie in the incidence plane.

At not too large intensities I, stimulated scattering is possible only in directions close to the values given by the Fresnel formulas. The maximum frequency Ω_0 of the amplified capillary waves is determined in this case by the equation

$$\Omega_0 = \frac{(\epsilon-1)I}{c\eta}\frac{k_x|k_z|}{\sqrt{\epsilon k^2 - k_x^2}}\frac{|k_z| - \sqrt{\epsilon k^2 - k_x^2}}{(|k_z| + \sqrt{\epsilon k^2 - k_x^2})^2}. \tag{6}$$

However, excessively small values of Ω_0 and q are not favorable for the observation of the stimulated scattering of light by surface waves. For this reason, in particular, we did not consider gravitational waves.

Estimates based on formula (6) show that for reasonable values of the frequency Ω_0 the intensity I cannot be attained by modern cw lasers. When pulsed lasers are used, the pulse duration τ is limited by the condition $\Omega_0\tau \gg 1$, with Ω_0 limited from above by the condition $q \lesssim k$. A numerical estimate by means of formula (5) with $\Omega_0 = 3 \times 10^7$ sec^{-1},

$\eta = 7 \times 10^{-4}$ poise, $\epsilon = 1.7$, and an incidence angle $\theta = \pi/6$ yields for the threshold intensity $I_0 = c\eta\Omega_0/4\pi|D|$ a value $I_0 \sim 8 \times 10^8$ W/cm^2. Such conditions can be realized, for example, for liquid nitrogen and a pulsed neodymium-glass laser operating in the free-running mode ($\tau \sim 10^{-3}$ sec).

[1] L. I. Mandel'shtam, Ann. Physik 41, 609 (1913).
[2] A. A. Andronov and M. Leontovich, Z. Physik 38, 485 (1926); A. A. Andronov, Collected Works, AN SSSR, 1956.
[3] R. H. Katyl and U. Ingard, Phys. Rev. Lett. 20, 248 (1968).
[4] L. D. Landau and E. M. Lifshitz, Mekhanika sploshnykh sred, Gostekhizdat, 1953 [Fluid Mechanics, Addison-Wesley, 1958].

OSCILLATIONS OF ULTRASOUND DAMPING IN A SEMICONDUCTOR IN A HIGH-FREQUENCY ELECTRIC FIELD

E. M. Epshtein
Institute of Radio Engineering and Electronics, USSR Academy of Sciences

Submitted 20 March 1968
ZhETF Pis'ma 7, No. 11, 433-436 (5 June 1968)

We show in this paper that the frequency dependence of ultrasound damping in a semiconductor has an oscillatory character in the presence of a homogeneous electric field whose frequency Ω is much larger than the frequency of the sound. The oscillations are "gigantic," i.e., their amplitude is of the same order of magnitude as that of the oscillating quantity itself *. We consider the case when the inequalities $q\ell \gg 1$, $\Omega\tau \gg 1$, and $\Omega \gg \omega_0$ are satisfied (q - wave vector of sound, ℓ and τ - mean free path and relaxation time of the electron, ω_0 - electron plasma frequency). For simplicity we confine ourselves to the case of an isotropic medium and a parabolic electron dispersion law. The complete system of equations describing the interaction of the electrons with the longitudinal ultrasonic wave and with the self-consistent field $\mathcal{E}$ is **

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}\,\nabla}{m} + e(\mathbf{E} + \boldsymbol{\mathcal{E}})\frac{\partial}{\partial \mathbf{p}} - \Lambda \nabla^2 u \frac{\partial}{\partial \mathbf{p}}\right) f = 0, \qquad (1)$$

$$\left(\frac{\partial^2}{\partial t^2} - s^2 \nabla^2\right) u = \frac{\Lambda}{\rho} \nabla \int f\, d^3p, \qquad (2)$$

$$\nabla \boldsymbol{\mathcal{E}} = -\frac{4\pi e}{\epsilon}\left(N - \int f\, d^3p\right), \qquad (3)$$

where u is the displacement in the sound wave, s the renormalized speed of sound, Λ the deformation-potential constant, ρ the crystal density, N the average electron density, $\vec{E} = \vec{E}_0 e^{\delta t} \sin \Omega t$ the external high-frequency field ($\delta \to +0$), m the effective mass of the electron, and ϵ the lattice dielectric constant. In the absence of sound or plasma oscillations

* It is assumed that the lattice absorption of the sound is much smaller than the electron absorption, or at least of the same order.

** We disregard the piezoelectric effect, since in the frequency region under consideration the main role is usually played by the deformation mechanism of the electron-phonon interaction.