$\eta = 7 \times 10^{-4}$ poise, $\epsilon = 1.7$, and an incidence angle $\theta = \pi/6$ yields for the threshold intensity $I_0 = c\eta\Omega_0/4\pi|D|$ a value $I_0 \sim 8 \times 10^8$ W/cm^2. Such conditions can be realized, for example, for liquid nitrogen and a pulsed neodymium-glass laser operating in the free-running mode ($\tau \sim 10^{-3}$ sec).

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OSCILLATIONS OF ULTRASOUND DAMPING IN A SEMICONDUCTOR IN A HIGH-FREQUENCY ELECTRIC FIELD

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We show in this paper that the frequency dependence of ultrasound damping in a semiconductor has an oscillatory character in the presence of a homogeneous electric field whose frequency Ω is much larger than the frequency of the sound. The oscillations are "gigantic," i.e., their amplitude is of the same order of magnitude as that of the oscillating quantity itself *. We consider the case when the inequalities $q\ell \gg 1$, $\Omega\tau \gg 1$, and $\Omega \gg \omega_0$ are satisfied (q - wave vector of sound, ℓ and τ - mean free path and relaxation time of the electron, ω_0 - electron plasma frequency). For simplicity we confine ourselves to the case of an isotropic medium and a parabolic electron dispersion law. The complete system of equations describing the interaction of the electrons with the longitudinal ultrasonic wave and with the self-consistent field $\mathcal{E}$ is **

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}\,\nabla}{m} + e(\mathbf{E} + \mathcal{E})\frac{\partial}{\partial \mathbf{p}} - \Lambda \nabla^2 \dot{u}\frac{\partial}{\partial \mathbf{p}}\right) f = 0, \tag{1}$$

$$\left(\frac{\partial^2}{\partial t^2} - s^2\nabla^2\right) u = \frac{\Lambda}{\rho}\nabla \int f\, d^3p, \tag{2}$$

$$\nabla \mathcal{E} = -\frac{4\pi e}{\epsilon}\left(N - \int f\, d^3p\right), \tag{3}$$

where u is the displacement in the sound wave, s the renormalized speed of sound, Λ the deformation-potential constant, ρ the crystal density, N the average electron density, $\vec{E} = \vec{E}_0 e^{\delta t} \sin \Omega t$ the external high-frequency field ($\delta \to +0$), m the effective mass of the electron, and ϵ the lattice dielectric constant. In the absence of sound or plasma oscillations

* It is assumed that the lattice absorption of the sound is much smaller than the electron absorption, or at least of the same order.

** We disregard the piezoelectric effect, since in the frequency region under consideration the main role is usually played by the deformation mechanism of the electron-phonon interaction.

($u = \delta = 0$), the solution of (1) takes the form

$$f^{(0)}(p,t) = f_0(p - \int_{-\infty}^{t} E(t')\,dt'),$$

where $f_0(\vec{p})$ is the equilibrium electron distribution function.

We shall seek a solution of the system (1) - (3) in the form $f(\vec{p}, r, t) = f^{(0)}(\vec{p}, t) + e^{i\vec{q}\cdot\vec{r}} f^{(1)}(\vec{p}, t)$, $\vec{u}(\vec{r}, t) = e^{i\vec{q}\cdot\vec{r}} \vec{u}^{(1)}(t)$, and $\delta(\vec{r}, t) = e^{i\vec{q}\cdot\vec{r}} \delta^{(1)}(t)$, carrying out the linearization with respect to the small quantities with index (1). Eliminating δ with the aid of (3), and introducing the new function

$$F(p,t) = \exp[\frac{ie}{m}\int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt''\, qE(t'')] f^{(1)}(p + e\int_{-\infty}^{t} E(t')dt', t),$$

we readily get ***

$$(\frac{\partial}{\partial t} + \frac{iqp}{m})F + \frac{4\pi e^2}{i\epsilon q^2} q \frac{\partial f_0(p)}{\partial p} \int F d^3p =$$

$$= -\Lambda q q \frac{\partial f_0(p)}{\partial p} u^{(1)} \sum_{n=-\infty}^{\infty} (-1)^n I_n(a) e^{in\Omega t}, \qquad (4)$$

$$(\frac{\partial^2}{\partial t^2} + s^2 q^2) u^{(1)} = \frac{i\Lambda q}{\rho} \int F d^3p \sum_{n=-\infty}^{\infty} I_n(a) e^{in\Omega t}, \qquad (5)$$

where $a = e\vec{E}_0\vec{q}/m\Omega^2$ and I_n is a Bessel function of zero order.

Let us average (4) and (5) over the period of the high-frequency field. Since we are considering only waves with frequencies much lower than the frequency of this field, to obtain the "abbreviated" equations it is sufficient to replace F and $u^{(1)}$ by their averaged values, and to retain in the right sides of the equations only the terms with $n = 0$. Assuming that the averaged quantities are proportional to $\exp(-i\omega t)$ $(\omega \ll \Omega)$, we get the dispersion equation

$$[1 + \frac{4\pi e^2}{\epsilon q^2} \int \frac{q(\partial f_0/\partial p)\,d^3p}{\omega - qp/m}](\omega^2 - s^2q^2) = $$
$$= -\frac{\Lambda^2}{\rho} I_0^2(a)\, q^2 \int \frac{q(\partial f_0/\partial p)\,d^3p}{\omega - qp/m}. \qquad (6)$$

It follows therefore that the effect of the high-frequency field is equivalent to a renormal-

*** Such a method was used by Aliev and Silin [1].

ization of the constant of the deformation potential $\Lambda \to \Lambda I_0(a)$. This seemingly small change leads to an interesting consequence. In fact, the sound absorption coefficient determined from the dispersion equation (6) has at sufficiently small damping the value

$$\alpha = \frac{2}{s} Im\, \omega = \sqrt{\frac{\pi}{2}}\, \Lambda^2 I_0^2(a)\, \frac{m^{1/2} N q}{\rho s (kT)^{3/2}} \left(\frac{q^2}{q^2 + \kappa^2}\right)^2, \tag{7}$$

where κ is the reciprocal of the screening radius and T the temperature of the crystal. Consequently, the electronic sound absorption coefficient oscillates at a function of the parameter a, vanishing at values of a corresponding to the zeroes of the Bessel function. The most interesting is the dependence of α on the frequency (or on the wave number) of the sound. Thus, when $q \gg \kappa$ and $\vec{q} \parallel \vec{E}_0$ we have

$$\alpha(q) \sim q I_0^2 \left(\frac{e E_0 q}{m \Omega^2}\right),$$

so that when $a \gg 1$ we have

$$\alpha(q) = \mathrm{const}\, \cos^2 \left(\frac{e E_0 q}{m\Omega^2} - \frac{\pi}{4}\right),$$

i.e., the $\alpha(q)$ dependence has a purely oscillatory character.

The predicted effect can be interpreted physically as a geometrical resonance between the amplitude of the electric oscillations in the high-frequency field $eE_0/m\Omega^2$ and the given sound wave $2\pi/q$. This phenomenon is analogous to geometrical resonance in a magnetic field (see, e.g., [2]), but the role of the electron revolution in the Larmor orbit is played in this case by the electron oscillations in the high-frequency field.

Measurement of the period of the oscillations of the sound damping can be used to determine the effective mass of the electron, and measurement of the amplitude can be used to separate the lattice and electronic contributions to the sound absorption.

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E R R A T A

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In formula (5) on p. 341 and in the line following it, "I_n" should be replaced by "J_n."

On p. 342, lines 16-17, "given sound wave $2\pi/q$" should be replaced by "sound wavelength $2\pi/q$."