

Let us stop to discuss the Mossbauer scattering of the  $\gamma$  radiation by a crystal with antiferromagnetic ordering of the magnetic fields at the Mossbauer nuclei (for example, a collinear antiferromagnet). In this case formula (2) for the scattering cross section takes the form

$$\frac{d\sigma}{d\Omega} = A^2 |f_{\uparrow} + f_{\downarrow} e^{i(\mathbf{k} - \mathbf{k}') \cdot (\mathbf{r}_{\uparrow} - \mathbf{r}_{\downarrow})}|^2 \sum_r \delta(\mathbf{k} - \mathbf{k}' - \mathbf{r}), \quad (5)$$

where  $\vec{\tau}$  is the vector of the reciprocal lattice of the magnetic structure, and the symbols  $(\uparrow, \downarrow)$  are introduced since there are only two mutually opposite non-equivalent direction of the magnetic field at the Mossbauer isotopes in this case. For the polarization vector of the scattered radiation, neglecting Rayleigh scattering, we can obtain from (5) (see (2) and (3))

$$\mathbf{n} = \frac{\mathbf{n}_0'(\mathbf{n} \cdot \mathbf{n}_0') + \mathbf{n}_0''(\mathbf{n} \cdot \mathbf{n}_0'')}{|\mathbf{n}_0'(\mathbf{n} \cdot \mathbf{n}_0') + \mathbf{n}_0''(\mathbf{n} \cdot \mathbf{n}_0'')|}, \quad (6)$$

where the upper sign pertains to the pure magnetic maxima and the lower one to the crystalline ones. We considered above only cases of fully coherent scattering. As is well known, allowance for factors that lead to a partial incoherence (crystal defects, crystal thermal vibrations, nonzero ground-state spin of the scattering nuclei, etc.) will not change the results qualitatively.

In conclusion, to illustrate the foregoing example of a collinear antiferromagnet, we present the values of the Bragg angles at which magnetic maxima occur for the scattering of Mossbauer radiation with energy 14.4 keV from Fe<sup>57</sup> by antiferromagnetic single-crystal Fe<sub>2</sub>O<sub>3</sub>. The first two crystal maxima are at the Bragg angles  $\sim 16$  and  $23^\circ$ , and the first two magnetic maxima at  $\sim 11$  and  $20^\circ$ .

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#### POSSIBLE VERIFICATION OF THE EQUIVALENCE PRINCIPLE (FOR ATOMIC PHENOMENA) IN THE SECOND AND HIGHER ORDERS IN THE NEWTONIAN GRAVITATIONAL CONSTANT

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Submitted 9 April 1968

ZhETF Pis'ma 7, No. 12, 481-484 (20 June 1968)

1. The purpose of the present paper is to show that violation of the equivalence principle in the relativistic equations for the electron (in the presence of a gravitational field) could lead to a new observable effect - anomalous red shift of the spectral lines of a source (hydrogen atom) situated in a gravitational field. The anomaly would consist in the fact that the coefficient  $(1 + z)$  of proportionality of the wavelength received from the indicated source to the corresponding laboratory wavelength would be different for different spectral lines (a similar phenomenon would occur if the fine structure constant at the source

were not the same as at the receiver) <sup>1)</sup>.

The measurements of Pound and Rebka [1], as well as many astronomical measurements of the red shift, give grounds for stating that no such anomaly exists in the first order in the Newtonian gravitational constant  $G$ . To verify whether this anomaly exists or not in the second and higher orders in  $G$ , we propose to carry out the spectroscopic measurements on light from quasars, where  $z \gtrsim 1$ , the more convenient objects for such an investigation would be quasars of gravitational (and not cosmological) origin. Thus, by performing spectroscopic measurements of light from quasars with large  $z$  one could greatly decrease the number of gravitational theories that are assumed at present to agree with experiment, i.e., with the data on the three classical tests.

2. By equivalence principle we mean the requirement that all the phenomena occurring in an infinitesimally small Einstein lift have the same form as the corresponding phenomena in the absence of gravitation and relative to an inertial reference frame. From this definition of the principle it follows, in particular, that no gravitational field should cause violation of the ordinary (laboratory) proportions between the wavelengths emitted by atoms situated in this gravitational field, since the violation or non-violation of the indicated proportions is invariant to the choice of the reference frame.

Thus, in any theory containing the equivalence principle for atomic phenomena, the system of equations leading to observable wavelengths should have some singularity capable of excluding the appearance of disproportions in the energy levels of the stationary states, in spite of the presence of gravitational potentials in this system of equations. For example, the general-covariant equations of the spinor field certainly do not lead to the red-shift anomaly [2].

3. By way of the simplest example of a theory leading to an anomalous gravitational red shift, let us consider the linear theory of gravitation [3-5], and also the Thirring non-linear theory which is close to it [6], as applied to the hydrogen atom or to a hydrogenlike ion. The Dirac equation with a gravitational field, which enters in these theories, was analyzed in detail by Moshinsky in [3]. It turns out, however, that the stationary levels can be calculated quite simply from this equation not only in the first order in the Newtonian constant (as was done in [3]), but also exactly:

$$E_{n',j'} = (1+f)^{-1} mc^2 \{1 + \tilde{\alpha}^2 [n' + \sqrt{j'^2 - \tilde{\alpha}^2}]^2\}^{-1/2}, \quad (1)$$

$$n' = 0, 1, \dots; j' = 1, 2, \dots,$$

where  $\tilde{\alpha} = (1+f)(1+f-2f^2)^{-1}\alpha$ ,  $\alpha = Ze^2/\hbar c$ ,  $Ze$  is the nuclear charge, and the remaining symbols are those used in [3]. The solution (1) is obtained if the renormalization of the constants employed in [3] is replaced by

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<sup>1)</sup> Since we confine ourselves here to a qualitative formulation of the question, we shall assume that the receiver is arbitrarily far away from the gravitating centers; it can be shown that allowance for the gravitational field of the Universe as a whole would only lead to additional effects, including simpler observable effects connected with the anomalous red shift.

$$\tilde{E} = (1 + f)E, \quad \tilde{e}^2 = (1 + f)(1 + 2f)^{-1}e^2, \quad \tilde{h} = (1 - f)h, \quad \tilde{m} = m, \quad \tilde{c} = c.$$

We expand the right side of (1) in powers of  $\alpha$ :

$$E_{nJ} = mc^2 \left\{ (1 + f)^{-1} + (1 + z)^{-1} \left[ \frac{a^2}{2n^2} + (1 + \xi)^2 \frac{a^4}{2n^3} \left( \frac{1}{J} - \frac{3}{4n} \right) + \dots \right] \right\} \quad (2)$$

where  $J = 1, 2, \dots$ ;  $n = j + 0, 1, 2, \dots$ ;  $\xi = \tilde{\alpha}/\alpha - 1$ ; and  $1 + z = (1 - f)^2(1 + f)^2(1 + f)^{-1}$ . It is obvious that the parameter  $z$  determines the mean relative lengthening (due to the red shift) of the observable wavelengths that are connected with transitions between levels having different values of the principal quantum number  $n$ , and the parameter  $\xi$  determines the gravitational distortion of the spectrum. The dependence of the parameters  $z$  and  $\xi$  on the gravitational potential  $f$  is shown in the table. For comparison, the fourth and fifth columns list the values of the parameters  $z'$  and  $\xi'$  obtained from the Moshinsky Lagrangian for a Dirac particle, and from the spinor-field Lagrangian linearized in the gravitational potentials, proposed by Ogievetskii and Polubarinov [2]. In our opinion it would be more consistent to assume this (linearized) Lagrangian in the linear theory of gravitation for the spinor field (in the presence of gravitation).

Table of values of the functions  $z(f)$ ,  $\xi(f)$ ,  $z'(f)$ , and  $\xi'(f)$

	$z$	$\xi$	$z'$	$\xi'$
0.00	0.000	0.000	0.000	0.000
0.0500	0.040	0.0048	0.055	-0.0043
0.125	0.063	0.029	0.15	-0.022
0.250	0.012	0.11	0.34	-0.067
0.375	-0.13	0.26	0.56	-0.12
0.500	-0.33	0.50	0.80	-0.17

We note that in sufficiently strong gravitational fields formula (2) leads not to a red but to a blue gravitational shift, unlike general relativity theory. The values  $f > 0.5$  were not considered by us, since at these values of  $f$  the electrodynamics of the theories in question admits of no solutions in wave form.

In conclusion, the author considers it his pleasant duty to thank Ya. A. Smorodinskii for help with the work and I. D. Nogikov for a discussion.

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