DAMPING OF PROPER WAVES IN A PLATE WITH ROUGH WALLS

F. G. Bass, V. D. Freilikher, and I. M. Fuks
Institute of Radiophysics and Electronics, Ukrainian Academy of Sciences
Submitted 17 April 1968
ZhETF Pis'ma 7, No. 12, 485-488 (20 June 1968)

The surface roughness of a thin film can exert a significant influence on the spectrum of the various quasiparticles (electrons, phonons, etc.), leading to a shift and broadening of their energy levels. This broadening of the spectrum is similar in nature to the decrease of the phase velocity and to the appearance of damping of normal waves propagating in a rough waveguide made up of two infinite surfaces $z = \zeta_1(\vec{r})$ and $z = a + \zeta_2(\vec{r})$, where $\vec{r} = \{x, y\}$, $\{x, y, z\}$ are the Cartesian coordinates, and $\zeta_1(\vec{r})$ and $\zeta_2(\vec{r})$ are random functions with zero mean value.

In our case, the problem reduces to a solution of a Helmholtz equation for the wave function ψ (or a scalar potential in the case of a waveguide)

$$\Delta\psi + k^2\psi = 0 \qquad (1)$$

with allowance for the concrete boundary conditions on the surfaces ζ_1 and ζ_2. We consider here the simplest case of zero Dirichlet boundary conditions, when $\psi(z = \zeta_1) = \psi(z = \zeta_2 + a) = 0$. For electrons, for example, such a boundary condition corresponds to an infinitely tall potential barrier on the film boundary; for phonons this means that the thin film is in vacuum or, in other words, is an acoustic waveguide with absolutely soft walls.

Expanding these boundary conditions in powers of ζ_1 and ζ_2 and performing the averaging $\langle\ldots\rangle$ over the ensemble of the realizations of ζ_1 and ζ_2, we obtain for the average field $\langle\psi\rangle$ and for the fluctuating component φ the following relations, which are satisfied when $z = 0$ and $z = a$:

$$\langle\psi\rangle + \left\langle \zeta\frac{\partial\phi}{\partial z}\right\rangle = 0; \quad \phi + \zeta\frac{\partial\langle\psi\rangle}{\partial z} = 0. \qquad (2)$$

With the aid of Green's formula we obtain from (2) the effective nonlocal boundary conditions for the average field at the planes $z = 0$ and $z = a$:

$$\langle\psi(\mathbf{R})\rangle\Big|_{z=0,a} = \pm\frac{\sigma^2}{4\pi}\int \frac{\partial^2 G_0(\mathbf{R},\mathbf{R}')}{\partial z\,\partial z'}\Big|_{z=z'=0,a} W(\mathbf{r},\mathbf{r}')\frac{\partial\langle\psi(\mathbf{R}')\rangle}{\partial z'}\Big|_{z'=0,a} d^2r' \qquad (3)$$

Here $\sigma^2 = \langle\zeta_1^2\rangle = \langle\zeta_2^2\rangle$; $\sigma^2 W(\vec{r}, \vec{r}') = \langle\zeta_1(\vec{r})\zeta_1(\vec{r}')\rangle = \langle\zeta_2(\vec{r})\zeta_2(\vec{r}')\rangle$; $\langle\zeta_1\zeta_2\rangle = 0$; $\vec{R} = \{\vec{r}, z\}$; $G_0(\vec{R}, \vec{R}')$ is the Green's function of a flat unperturbed waveguide with absolutely soft walls. The plus sign corresponds to $z = 0$ and the minus sign to $z = a$. Formulas (3) are valid only for the case of the so-called noncritical frequencies, i.e., those for which

$$\frac{ak}{\pi} = N + \epsilon,\ N = \left[\frac{ak}{\pi}\right]$$

ϵ is the noncriticality parameter; $0 < \epsilon < 1$. If $\epsilon = 0$, then the width of the waveguide spans an integer number of half waves, and G_0 becomes infinite.

We assume further for simplicity that $\zeta_i = \zeta_i(x)$, and consider the two-dimensional Helmholtz equation. Then the solution (1) with boundary conditions (3) can be written in the form

$$\langle\psi(x, z)\rangle = (c_1 e^{iqz} + c_2 e^{-iqz}) e^{ix\sqrt{k^2 - q^2}}. \tag{4}$$

Substituting (4) in (3) we obtain a homogeneous system of equations for the determination of c_1 and c_2. The requirement that its determinant vanish leads to the following dispersion equation for the determination of the eigenvalues q_n:

$$\delta q_n = q_n^0 \frac{k\sigma^2}{a\pi} \int_{-\infty}^{\infty} \sqrt{1-t^2}\, \mathrm{ctg}(ak\sqrt{1-t^2})\, \widetilde{W}(t+a_n)\, dt - \\ - \frac{i\sigma^2\pi^2}{a^4 k^2} q_n^0 \sum_{\nu=1}^{N} \frac{\nu^2}{\beta_\nu} [\widetilde{W}(\beta_\nu + a_n) + \widetilde{W}(\beta_\nu - a_n)], \quad n = 1, 2 \ldots \tag{5}$$

Here $q_n^0 = n\pi/a$ are the eigenvalues of the unperturbed waveguides, and the corrections to these values $\delta q_n = q_n - q_n^0$ are assumed to be small compared with the distance between levels $|\delta q_n| \ll q_n^0 - q_{n-1}^0$; $\widetilde{W}(t)$ is the Fourier transform of the correlation function; $\beta_\nu = \sqrt{1 - (\nu^2\pi^2/a^2k^2)}$, and $a_n = \sqrt{1 - (q_n^2/k^2)}$.

We see that all the δq_n have imaginary parts, i.e., even those waves which propagate in a smooth waveguide (and for which $n\pi/ak < 1$) acquire a finite damping in the presence of the boundary perturbation (the quantum levels of the quasiparticles broaden). The form of the sum entering in (5) shows that this damping is due to the transformation of a wave with a given number n into all the remaining propagating modes. The level shift ($\mathrm{Re}\,\delta q_n$) is due to transformation into inhomogeneous (non-propagating) waves.

If the correlation radius l is large compared with the wavelength ($kl \gg 1$), then $\widetilde{W}(t)$ is a "sharp" function. If the condition $l \gg \Lambda_n$ is satisfied, where $\Lambda_n = a(q_n^0/\beta_n) = a \tan \psi_n$ is the length of the cycle of the given mode, i.e., the distance between two successive reflections (ψ_n - glancing angle), the only important term in the sum (5), regardless of the value of N, is the term with $\nu = n$. Then, for example for a Gaussian correlation function, the solution of the dispersion equation takes the form

$$\delta q_n = \frac{k\sigma \sin\psi_n}{a\sqrt{2\pi}} - iO(e^{-(k\ell)^2}) \quad \text{if} \quad \frac{\sigma^2}{a^2} k\ell \gg 1, \\ \delta q_n = \frac{k\sigma^2 \sin^3\psi_n}{a^2\cos^2\psi_n}[1 - i\cos\psi_n \sqrt{\pi} k\ell] \quad \text{if} \quad \frac{\sigma^2}{a^2} k\ell \gg 1. \tag{6}$$

When a multimode waveguide is considered, $N \gg 1$ (film thickness much larger than the wavelength), the dispersion equation also simplifies considerably. In this case, if $l \ll \Lambda_n$, the sum in (5) can be replaced by an integral, as a result of which we get

$$\delta q_n = -i\,\frac{1 + V(\psi_n)}{a}, \tag{7}$$

where $V(\psi_n)$ is the coefficient of reflection of the plane wave from a half-space bounded by a rough plane [1].

We note that formula (7) can be obtained by calculating $\langle\psi_n(\vec{R})\rangle$ as a result of successive reflections of a normal wave with an effective reflection coefficient $V(\psi)$ [2,3]. It is seen from the foregoing that such a method is suitable only for sufficiently broad waveguides ($ak \gg 1$), it being necessary that the correlation radius of the roughnesses be small compared with the length of the cycle ($l \ll \Lambda_n$).

Owing to the approximate character of the effective boundary conditions (3), formula (7) describes the damping of the average field only at distances satisfying the inequalities $L(k^2\sigma^4/al)q_n^0 \ll 1$ in the case when $kl \ll 1$ and $L[(k\sigma)^4/a]\sin^4\psi_n \ll 1$ if $kl \gg 1$. We see that if $(\sigma/l)^2 \ll 1$ (when $kl \ll 1$) or $(k_z\sigma)^2 \ll 1$ (when $kl \gg 1$) these distances greatly exceed the length $L_{eff} \sim (\mathrm{Im}\ \delta q_n)^{-1}$ whitin which the average field attenuates.

[1] F. G. Bass, Izv. Vuzov, Radiofizika 4, 476 (1961).
[2] Yu. P. Lysanov, Akust. Zh. 12, 489 (1966) [Sov. Phys.-Acoust. 12, 425 (1967)].
[3] C. S. Clay, J. Acoust. Soc. Amer. 36, 833 (1964).

CYCLOTRON RESONANCE IN THIN FILMS

V. G. Peschanskii
Physico-technical Institute of Low Temperatures, Ukrainian Academy of Sciences

Submitted 23 April 1968; resubmitted 12 May 1968
ZhETF Pis'ma 7, No. 12, 489-492 (20 June 1968)

Nee and Prange [1] have shown that a quantum resonance can be observed in investigations of the high-frequency properties of conductors in weak magnetic fields (Larmor radius r larger than the electron mean free path l). This resonance is due to electrons that remain practically trapped in a narrow skin layer of thickness δ as a result of specular collisions with the surface of the sample. The distance between the quantized energy levels of such electrons turns out to be large enough to resolve the resonant impedance peaks even in a weak magnetic field. This is how Nee and Prange explain the impedance oscillations observed in 1960 by Khaikin in bulky samples of bismuth and tin [2].

In a weak magnetic field parallel to the surface of such a plate (of thickness $d \ll l$) there can occur also a classical resonance effect, connected with the fact that the electron specularly reflected from the opposite surface of the plate can frequently fall into the skin layer. The electron thus moves along an open periodic orbit and interacts in resonant fashion with the electromagnetic wave, if the latter is launched in the skin layer at the same phase, i.e., the period of motion of the electron T_λ satisfies the condition

$$\omega T_\lambda(p_z) = 2\pi n. \tag{1}$$