the sum in (5) can be replaced by an integral, as a result of which we get

$$\delta q_n = -i\frac{1 + V(\psi_n)}{a}, \tag{7}$$

where $V(\psi_n)$ is the coefficient of reflection of the plane wave from a half-space bounded by a rough plane [1].

We note that formula (7) can be obtained by calculating $\langle\psi_n(\vec{R})\rangle$ as a result of successive reflections of a normal wave with an effective reflection coefficient $V(\psi)$ [2,3]. It is seen from the foregoing that such a method is suitable only for sufficiently broad waveguides ($ak \gg 1$), it being necessary that the correlation radius of the roughnesses be small compared with the length of the cycle ($\ell \ll \Lambda_n$).

Owing to the approximate character of the effective boundary conditions (3), formula (7) describes the damping of the average field only at distances satisfying the inequalities $L(k^2\sigma^4/a\ell)q_n^0 \ll 1$ in the case when $k\ell \ll 1$ and $L[(k\sigma)^4/a]\sin^4\psi_n \ll 1$ if $k\ell \gg 1$. We see that if $(\sigma/\ell)^2 \ll 1$ (when $k\ell \ll 1$) or $(k_z\sigma)^2 \ll 1$ (when $k\ell \gg 1$) these distances greatly exceed the length $L_{eff} \sim (\mathrm{Im}\ \delta q_n)^{-1}$ whitin which the average field attenuates.

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CYCLOTRON RESONANCE IN THIN FILMS

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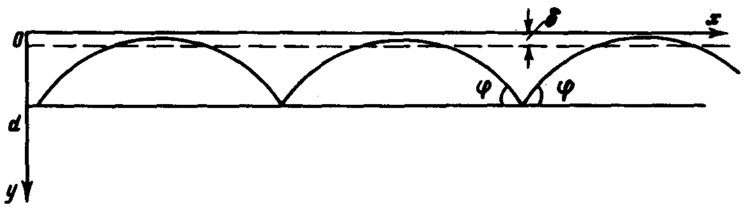
Nee and Prange [1] have shown that a quantum resonance can be observed in investigations of the high-frequency properties of conductors in weak magnetic fields (Larmor radius r larger than the electron mean free path ℓ). This resonance is due to electrons that remain practically trapped in a narrow skin layer of thickness δ as a result of specular collisions with the surface of the sample. The distance between the quantized energy levels of such electrons turns out to be large enough to resolve the resonant impedance peaks even in a weak magnetic field. This is how Nee and Prange explain the impedance oscillations observed in 1960 by Khaikin in bulky samples of bismuth and tin [2].

In a weak magnetic field parallel to the surface of such a plate (of thickness $d \ll \ell$) there can occur also a classical resonance effect, connected with the fact that the electron specularly reflected from the opposite surface of the plate can frequently fall into the skin layer. The electron thus moves along an open periodic orbit and interacts in resonant fashion with the electromagnetic wave, if the latter is launched in the skin layer at the same phase, i.e., the period of motion of the electron T_λ satisfies the condition

$$\omega T_\lambda(p_z) = 2\pi n. \tag{1}$$



Here ω is the frequency of the electromagnetic field, n is an integer, p_z is the projection of the electron momentum on the direction of the constant magnetic field, and the choice of the remaining coordinate axes is clear from the figure.

To observe the resonance it is necessary that the electron whose period T_λ is extremal with respect to p_z enter many times in the skin layer during the mean free path time t_0, i.e.,

$$T_\lambda(p_z^0) << t_0; \quad \frac{\partial T_\lambda}{\partial p_z}\bigg|_{p_z = p_z^0} = 0. \tag{2}$$

The period of the electron motion T_λ, which is equal to the difference between the roots λ of the equation

$$p_x(p_z, \lambda) - p_x^{min}(p_z) = \frac{eHd}{c},$$

is determined in a weak magnetic field essentially by the characteristics of the electron at the point of the stationary phase (at the point on the orbit where $v_y = 0$), i.e.,

$$T_\lambda(p_z^0) = \frac{\sqrt{8r_0 d}}{v_{0x}}, \tag{3}$$

where r_0 is the radius of curvature of the trajectory, $p_z = p_z^0$ at the stationary-phase point, v_0 is the electron velocity at the same point, e is the electron charge, c is the speed of light, and λ is the time of motion of the electron in the magnetic field.

Using (3), we obtain the following expression for the frequencies at which resonance takes place

$$\omega = \pi n \left(\frac{eH}{cd}\right)^{1/2} \left(\frac{v_{0x}}{\sqrt{2p_0}}\right)_{extr}; \quad p_0 \equiv \frac{eHr_0}{c}. \tag{4}$$

This resonance effect recalls the cyclotron resonance of Azbel' and Kaner [3] and differs from it only in the mechanism whereby the electron is returned to the skin layer. In a bulky sample cyclotron resonance is possible only in strong magnetic fields $r << l$, and in a thin plate the frequent return of the electrons to the skin layer is enhanced by their specular reflection from the surface $y = d$. The criterion for the "strong" magnetic field is therefore different. Taking (3) into account, condition (2) for a thin film becomes

$$r \ll l^2/d, \tag{5}$$

i.e., the limitation on the magnetic field intensity is less stringent than in bulky samples, and cyclotron resonance is possible also in weak magnetic fields $r > l$.

An exact calculation leads to the same results. The complete system of equations of the problem consists of Maxwell's equations and the Boltzmann kinetic equation. In place of the boundary condition on the surface $y = d$ (the condiction for specular reflection of the electrons) it is possible to introduce a fictitious Fermi surface, the intersections of which with the plane p_z = const are open periodic trajectories. Since only the electrons that do not collide with the surface $y = 0$ play any role in the resonance, it is possible to continue the electric field and the current in even fashion into the region $y < 0$, after which the problem is easily solved by the Fourier method. We present the final expression for the resonant part of the impedance:

$$Z^{res}_{\alpha\beta} = \dot{Z}^{0}_{\alpha\beta} A_{\alpha\beta}\{1 - \exp[-(i\omega + \frac{1}{t_0})T^{extr}_{\lambda}]\}^{1/6}; \ \alpha, \beta = (x, z), \tag{6}$$

where $Z^0_{\alpha\beta}$ is the impedance in the absence of a magnetic field, $A_{\alpha\beta} \sim v_{0\alpha}v_{0\beta}/v_0^2$ is a constant factor, and g is the parameter of the specularity of the electron reflection from the surface of the sample. The impedance is minimal when $H = H_{res}$, where

$$H_{res} = \frac{2p_0}{v^2_{0x}} \frac{c\,\omega^2 d}{\pi^2 e n^2}, \tag{7}$$

and the form of the resonance curve is determined not only by the mean free path of the electron, but also by the thickness of the sample and the parameter of the specularity of the electron scattering:

$$Z^{res}_{\alpha\beta} \simeq Z^0_{\alpha\beta} A_{\alpha\beta}(q_1 + \sqrt{r_0 d/l} + i\,\Delta)^{1/6}; \quad q_1 = 1 - q; \ \Delta = \frac{H - H_{res}}{H_{res}}, \tag{8}$$

The resonant peaks are periodic in the variable $H^{-1/2}$ and are most intense when the high-frequency electric field is polarized along the x axis. Knowing H_{res} we can determine the extremal value of the quantity $v_{0x}/\sqrt{p_0}$, and from the smearing of the resonance curve we can estimate the parameter of the specularity of the scattering of the electrons by the surface of the sample. If several resonance peaks can be resolved, then the parameter q as a function of the electron scattering angle $2\varphi \simeq (d/r_0)^{1/2}$.

To observe cyclotron resonance in a thin plate, just as in a bulky sample, it is necessary to have high electromagnetic field frequencies, $\omega t_0 \gg 1$, and strict parallelism of the magnetic field and the surface of the plate (the field inclination must not exceed δ/l). The thickness of the plate should oscillate within the same range of δ/l, and the lower surface of the plate $y = d$ must be sufficiently smooth to ensure specular reflection of the electrons.

If the electron free path is $l \simeq 0.1 - 1$ cm, then, for example in a bismuth plate $d \sim 10^{-3}$ cm thick, one can expect cyclotron resonance at microwave frequencies in magnetic fields $H \simeq 0.01 - 1$ Oe. The Khaikin oscillations are also observed in the same region of magnetic fields if the electrons are specularly scattered from the surface $y = 0$. In a thin plate with only one surface that scatters the electrons specularly, each of these effects can be observed separately by reversing the roles of the plate surfaces.

There is no doubt whatever that electrons in semimetals are scattered practically specularly from the surface of the sample [4].

In metals, the surface of the sample scatters specularly at least the "resonance" electrons, since their scattering angle is small, but this suffices for the observation of cyclotron resonance in a weak magnetic field.

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CRITICAL APERTURE IN PLASMA DIAGNOSTICS BY THE SCATTERING METHOD

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In plasma diagnostics by the method of scattering of laser emission from free electrons, the plasma parameters are determined by analyzing the density distribution of the scattering spectrum. The density of the spectrum is proportional to the scattering cross section σ and to the solid angle $\Delta\Omega$ in which the scattered light is gathered.

The laser emission frequency usually greatly exceeds the collision frequency of the charged particles of a laboratory plasma. Therefore the weak scattering of the electrons in the Coulomb field of the ions prevails in the interaction between the electrons and the ions. In this case, the scattering cross section for the electron component (in a plane perpendicular to the electric vector of a plane-polarized wave) is proportional to the function $\Gamma_\alpha(x)$ [1]:

$$\Gamma_a(x) = \frac{e^{-x^2}}{[1 - a^2(2xe^{-x^2}\int_0^x e^{t^2}dt - 1)]^2 + 4_\pi a^4 x^2 e^{-2x^2}}, \tag{1}$$

$$a = \sqrt{\frac{n_e e^2}{4\pi kT}}\,\frac{\lambda_0}{\sin\frac{\theta}{2}}, \tag{2}$$