magnetic field and the surface of the plate (the field inclination must not exceed δ/l). The thickness of the plate should oscillate within the same range of δ/l, and the lower surface of the plate $y = d$ must be sufficiently smooth to ensure specular reflection of the electrons.

If the electron free path is $l \simeq 0.1 - 1$ cm, then, for example in a bismuth plate $d \sim 10^{-3}$ cm thick, one can expect cyclotron resonance at microwave frequencies in magnetic fields $H \simeq 0.01 - 1$ Oe. The Khaikin oscillations are also observed in the same region of magnetic fields if the electrons are specularly scattered from the surface $y = 0$. In a thin plate with only one surface that scatters the electrons specularly, each of these effects can be observed separately by reversing the roles of the plate surfaces.

There is no doubt whatever that electrons in semimetals are scattered practically specularly from the surface of the sample [4].

In metals, the surface of the sample scatters specularly at least the "resonance" electrons, since their scattering angle is small, but this suffices for the observation of cyclotron resonance in a weak magnetic field.

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CRITICAL APERTURE IN PLASMA DIAGNOSTICS BY THE SCATTERING METHOD

L. N. Pyatnitskii, G. P. Khaustovich, and V. V. Korobkin
G. M. Krzhizhanovskii Power Engineering Institute

Submitted 24 April 1968
ZhETF Pis'ma 7, No. 12, 493-496 (20 June 1968)

In plasma diagnostics by the method of scattering of laser emission from free electrons, the plasma parameters are determined by analyzing the density distribution of the scattering spectrum. The density of the spectrum is proportional to the scattering cross section σ and to the solid angle $\Delta\Omega$ in which the scattered light is gathered.

The laser emission frequency usually greatly exceeds the collision frequency of the charged particles of a laboratory plasma. Therefore the weak scattering of the electrons in the Coulomb field of the ions prevails in the interaction between the electrons and the ions. In this case, the scattering cross section for the electron component (in a plane perpendicular to the electric vector of a plane-polarized wave) is proportional to the function $\Gamma_\alpha(x)$ [1]:

$$\Gamma_a(x) = \frac{e^{-x^2}}{[1 - a^2(2xe^{-x^2}\int_0^x e^{t^2}dt - 1)]^2 + 4_\pi a^4 x^2 e^{-2x^2}}, \qquad (1)$$

$$a = \sqrt{\frac{n_e e^2}{4\pi kT}}\,\frac{\lambda_0}{\sin\frac{\theta}{2}}, \qquad (2)$$

$$\omega_e = \frac{4\pi}{\lambda_0}\sqrt{\frac{2kT_e}{m}}\sin\frac{\theta}{2}. \qquad (3)$$

$x = \omega/\omega_e$ is the dimensionless frequency, $\omega = \omega' - \omega_0$ is the shift of the frequency ω' relative to ω_0, ω_0 and λ_0 are the frequency and wavelength of the laser emission, k is Boltzmann's constant, and T_e is the electron temperature.

The character of the spectrum is determined by the parameter α (see Fig. 1). In the absence of collective effects in the plasma ($\alpha \ll 1$) the spectrum does not depend on α and is described by a Gaussian curve, the characteristics of which make it possible to calculate n_e and T_e without particular difficulty. The influence of the collective effects ($\alpha \gtrsim 1$) becomes manifest in the appearance of satellites in the spectra and makes the cross section a function of α, and consequently also of the scattering angle θ. This means that within the limits of the observation angle $\Delta\Omega$ the scattered light pertains to different scattering angles θ, i.e., to different α. This causes apparatus "broadening" of the satellite. Therefore (1) is valid only for sufficiently small angles $\Delta\Omega$. Yet the tendency in experimental practice is to use for the observation of the scattered light as large a solid angle as possible, amounting usually to 0.3 sr, but reaching also 1 sr, which undoubtedly tends to exaggerate the half-widths of the satellites, and consequently of the temperature.

To determine the critical values of the solid angle of observation $\Delta\Omega_{cr}$ at which the satellite broadening can be neglected, we note that it is necessary to satisfy the condition

$$\frac{\Delta(\delta\omega) + 2(\Delta\omega_1)}{\delta\omega} \leqslant \epsilon, \qquad (4)$$

where $\Delta(\delta\omega)$ is the change in the satellite half-width due to the direct influence of α on the half-width itself, $\Delta\omega_1$ is the broadening due to the shift of the satellite, and ϵ is a small number. If α is sufficiently large ($\alpha > 1$), then, as follows from Fig. 1, the broadening is due essentially to the shift, and therefore condition (4) simplifies to

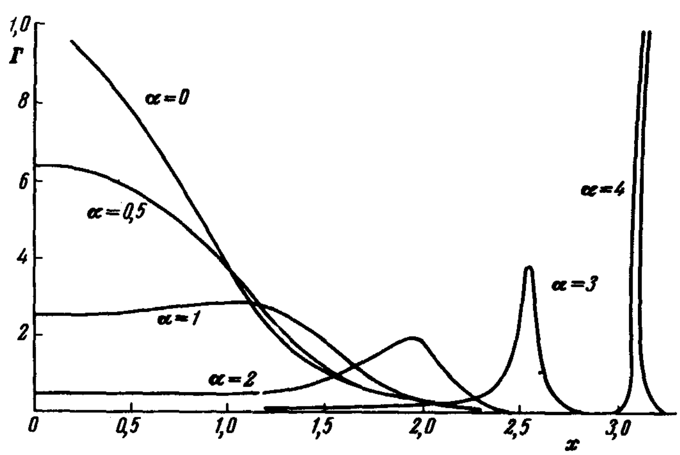

Fig. 1

$$\Delta\omega_1 \leq (\epsilon/2)\delta\omega. \tag{5}$$

The theory of [1] makes use not of the absolute frequencies, but of their relative values δx and x_1, which can be expressed in the form of monotonic functions:

$$\delta x = f(\alpha), \quad x_1 = (\alpha). \tag{6}$$

However, by definition $x = \Delta\omega/\omega_e$, and therefore

$$\Delta\omega_1 = x_1\omega_e, \quad \delta\omega = \delta x\omega_e. \tag{7}$$

According to (3), ω_e depends on θ, and therefore when ω_1 and $\delta\omega$ are replaced in (5) by x_1 and δx, the quantities ω_e do not cancel out, and what is indeed cancelled out is only a factor that does not depend on θ and characterizes the state of the plasma which, naturally, does not depend on the scattering angle. Making this replacement and cancelling, we obtain in lieu of (5)

$$\Delta\left(x_1 \sin\frac{\theta}{2}\right) \leqslant \frac{\epsilon}{2}\,\delta x \sin\frac{\theta}{2}, \tag{8}$$

or, with allowance for (6),

$$\Delta\left[\phi(a)\sin\frac{\theta}{2}\right] \leqslant \frac{\epsilon}{2} f(a) \sin\frac{\theta}{2}. \tag{9}$$

In differentiating the left side of (9) it must be borne in mind that α also contains two factors, one characterizing the plasma and independent of the aperture, and the other dependent only on the scattering angle. Therefore

$$\frac{\partial a}{\partial\theta} = -\frac{a}{2\,\mathrm{tg}\,\frac{\theta}{2}}. \tag{10}$$

Carrying out the corresponding differentiation, we obtain in place of (9):

$$\frac{1}{2}\cos\frac{\theta}{2}\left(x_1 - a\frac{\partial x}{\partial a}\right)\Delta\theta \leqslant \frac{\epsilon}{2}\,\delta x \sin\frac{\theta}{2}. \tag{11}$$

Finally, the condition for the critical values of the angle $\Delta\Omega_{cr}$ is

$$\Delta\Omega_{cr} = 2\pi\left[1 - \cos\left(\frac{\epsilon f(a)\,\mathrm{tg}\frac{\theta}{2}}{\phi(a) - a\frac{\partial\phi(a)}{\partial a}}\right)\right]. \tag{12}$$

Figure 2 shows plots of $\Delta\Omega_{cr}$ and α in the case $\varepsilon = 0.1$ for different scattering angles (curves 1, 2, 3, and 4 correspond to angles θ = 135, 90, 45, and 10°). A similar calculation can be carried out for the broadening of the satellite due to $\Delta(\delta\omega)$. In this case we obtain

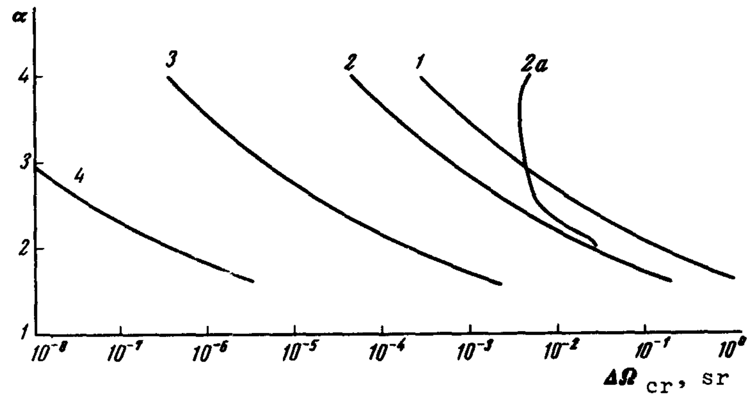

Fig. 2

for $\Delta\Omega_{cr}$ the expression

$$\Delta\Omega_{cr} = 2\pi\left[1 - \cos\left(\frac{2\epsilon f(a)\,\mathrm{tg}\,\frac{\theta}{2}}{f(a) - a\frac{\partial f(a)}{\partial a}}\right)\right]. \tag{13}$$

A curve of this type, delimiting the region of apertures, is also given in Fig. 2 (curve 2a) for a scattering angle $\theta = 90°$ and $\varepsilon = 0.1$.

The calculation shows that the critical values of $\Delta\Omega$ are much lower than the observation angles usually encountered in experiments. This in particular may explain why the temperatures calculated from the characteristics of the spectra are much higher than expected [2 - 4].

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Article by L. N. Pyatnitskii et al., Vol.7, No. 12

Formula (1) on p. 378 is in error. The corrected formula reads

$$\Gamma_\alpha(x) = e^{-x^2}\{[1 - \alpha^2(2xe^{-x^2}\int_0^x e^{t^2}dt - 1]^2 + \pi\alpha^4x^2e^{-2x^2}\}^{-1}.$$