NONLINEAR INTERACTION OF ION-CYCLOTRON WAVES IN THE MAGNETOSPHERE

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Nonlinear interaction of ion-cyclotron waves in the magnetosphere, which is manifest in a periodic exchange of energy between the waves, was observed and investigated experimentally.

Much attention is being paid of late [1 - 4] to the study of nonlinear phenomena occurring when monochromatic VLF and ULF waves propagate in the magnetosphere. It was shown, in particular [1, 3], that the growth of such waves in an unstable magnetospheric plasma can be accompanyed by production of satellites that are shifted in frequency, relative to the frequency of the fundamental wave, by an amount $\Delta f_s$ on the order of the characteristic period of the resonant-particle velocity oscillations.

The purpose of this study was to investigate experimentally nonlinear interaction between the satellites and the initial wave when ULF oscillations of the "pearl" type propagate in the magnetosphere. These oscillations are packets of ion-cyclotron waves in the 1 Hz range, which become amplified in the equatorial region of the magnetosphere and pass many times along the force line of the geomagnetic field, being reflected from the conjugated ionosphere.

Terrestrial observations of the oscillations with variational magnetometers were subjected to a spectral-temporal computer analysis with the aid of narrow-band mathematical filters

$$u(t) = \int_{-\infty}^{\infty} S(\omega)\exp\left\{-\alpha\left(\frac{\omega-\omega_k}{\omega_k}\right)^2 + i\omega t\right\}d\omega . \qquad (1)$$

Here $S(\omega)$ is the spectrum of the signal under consideration, $\omega_k$ is the central frequency of the filter, and $\alpha$ is the slope of the filter. The filter slope is chosen such that when the deviation from the central filter frequency is $\Delta f = 0.02$ Hz the signal amplitude is decreased by a factor e. Such an analysis makes it possible to separate the satellite from the original wave and to trace the simultaneous change of the amplitude of the envelope of two waves with a time resolution $\Delta t \simeq 50$ sec.

As a rule, satellite production becomes avalanche-like (each successive satellite becomes a source of production of other satellites). To separate the interaction of the satellite with the fundamental wave, we chose time intervals in which only two waves were present.

The computer-reduction data show the time variation of the signal envelope u(t) at each of the filtered frequencies. Figure 1 is typical spectral-temporal analysis of a series of "pearls," and shows packets of Alfven waves arriving in succession at one of the conjugate points, with a repetition period T = 150 sec. The degree of blackening characterizes the relative value of maximum signal amplitude.

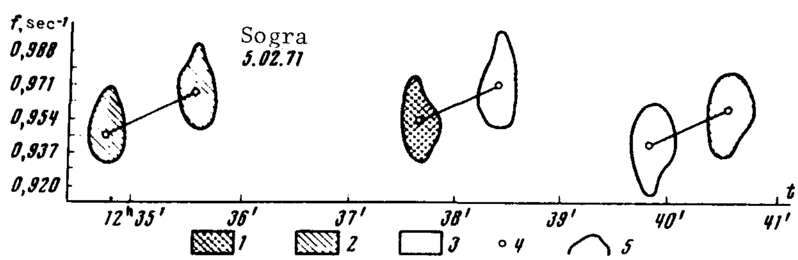


Fig. 1. Spectral-temporal analysis of a series of "pearls": 1 - $H^2_{max}/f > 25$ m$\gamma^2$/sec, 2 - $H^2_{max}/f > 10$ m$\gamma^2$ sec, 3 - $H^2_{max}/f > 3$ m$\gamma^2$ sec, 4 - maximum signal amplitude $H_{max}$, 5 - signal equal-amplitude line at the level $H = H_{max}e^{-1/2}$.

Figure 2 shows the time variation of the maximum amplitude of the packet for a fundamental wave $f_0 = 0.78$ Hz (solid lines) and for the satellite $f_s = 0.75$ Hz (dashed line). It is seen from Fig. 2 that after the fundamental wave reaches a sufficiently large amplitude, a satellite appears. The amplitude of the fundamental wave then decreases. The reason

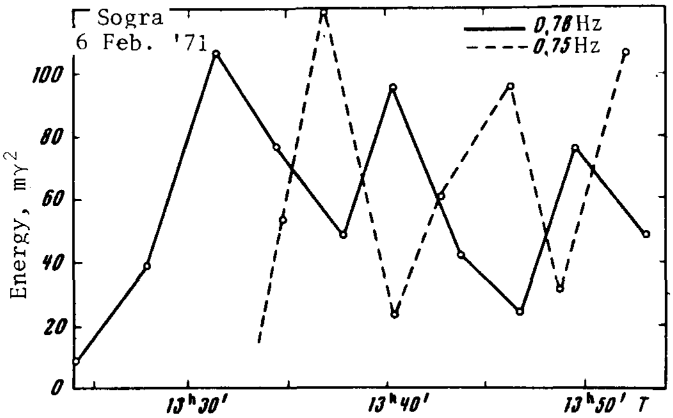


Fig. 2. Time variation of the amplitudes of the fundamental and of the satellite ($\Delta f = 0.03$ Hz).

for the latter is that when the satellite grows it draws energy from the fundamental wave. The next stage in the evolution is a successive energy exchange between the fundamental wave and the satellite, with a characteristic period ($\tau \simeq (2 - 3)T$ for the case considered here).

We analyzed altogether 15 cases in which the aforementioned periodic energy exchange between two waves has been observed. An analysis has shown that whereas in the case of small difference $f_s$ 0.02 Hz between the frequencies of the fundamental wave and the satellite the energy exchange is quite rapid, $\tau \simeq (2 - 3)T$, at large differences ($\Delta f_s \simeq 0.16$ Hz) the energy exchange is slow, $\tau \approx 8T$ (Fig. 3). The dependence of the characteristic period $\tau$ of energy exchange on the frequency shift $\Delta f_s$ is shown in Fig. 4 for the cases investigated in the paper. The period shows a tendency to increase with increasing $\Delta f_s$.

The situation described above recalls qualitatively the behavior of a system of coupled oscillators. At small detunings (corresponding to strong coupling) the energy exchange is fast enough, and vice versa.

It should be noted that for a quantitative description of the indicated effect it is necessary to construct a very complicated numerical model.

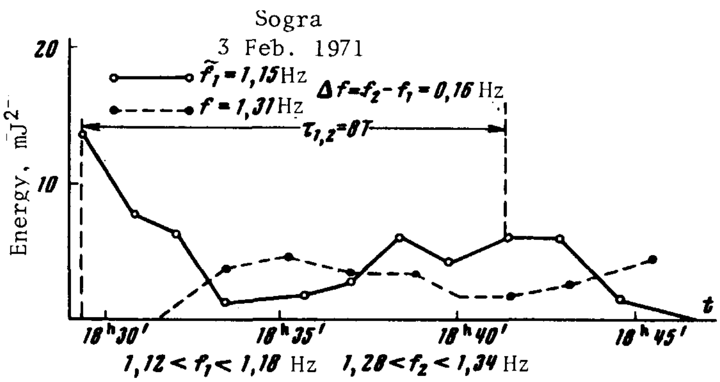


Fig. 3

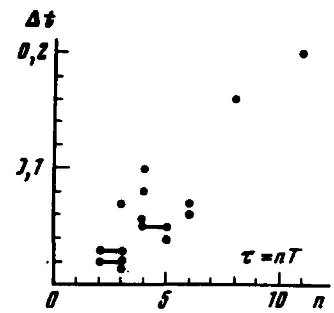


Fig. 4

Fig. 3. Time variation of the satellite and fundamental-wave amplitudes ($\Delta f = 0.16$ Hz).

Fig. 4. Dependence of the characteristic period of energy exchange between satellites and the fundamental wave on the frequency shift between them.


[1] N. I. Bud'ko, V. I. Karpman, and O. A. Pokhotelov, ZhETF Pis. Red. 14, 469 (1971) [JETP Lett. 14, 320 (1971)].

[2] M. B. Gokhberg, V. I. Karpman, and O. A. Pokhotelov, Dokl. Akad. Nauk SSSR 204, 848 (1972).

[3] N. I. Bud'ko, V. I. Karpman, and O. A. Pokhotelov, Cosmic Electrodynamics 3, 147 (1972).

[4] N. Wehrlin, R. Gendrin, A. Roux, and R. Welti, J. Geophys. Rev. 78, 543 (1973).