MAGNETIC PARAMETRIC RESONANCE IN CONDUCTORS

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A new resonance effect — magnetic parametric resonance (MPR) — is predicted for semiconductors placed in a magnetic field $\vec{H}$ that varies periodically with time and in a constant electric field $\vec{E}$ perpendicular to $\vec{H}$.

We consider an electron gas placed in an alternating homogeneous magnetic field $\vec{H} \parallel OZ$ and a constant electric field E. The value of H depends periodically on the time, with a frequency γ,

$$H = H_0(1 + \alpha \cos \gamma t), \quad \alpha = \text{const}. \tag{1}$$

The magnetic field is homogeneous in the conductor if the skin-layer depth $\delta\gamma$ at the frequency γ greatly exceeds the mean free path ℓ and the sample thickness d, i.e.,

$$\delta_\gamma = \left(\frac{c^2}{2\pi\sigma\gamma}\right)^{1/2} >> \ell, d; \qquad \sigma = \frac{Ne^2\ell}{mv}. \tag{2}$$

Here σ is the static conductivity, N the concentration, ℓ the absolute value of the charge, m the mass, and v the velocity of the conduction electrons. In addition, we assume that the amplitude E_γ of the induced electric field is small in comparison with the constant electric field E

$$E_\gamma \sim H_0 \alpha \frac{\gamma\delta_\gamma}{c} << E. \tag{3}$$

It is well known that in a constant and homogeneous magnetic field the motion of an electron in a plane perpendicular to the vector $\vec{H}_0$ is analogous to the behavior of a linear oscillator with constant cyclotron frequency $\Omega_0 = eH_0/mc$. The electron trajectory is described in this case by the integrals of motion ε = const and p_z = const, where ε is the energy and p_z is the projection of the electron momentum on the magnetic-field direction. In the case of an alternating magnetic field (1) (neglecting the field E_γ (3)), the trajectory is determined by the same conserved quantities, in contrast to the situation considered in [1, 2]. In other words, although the parameter of the system (the cyclotron frequency) varies with time, the system remains conservative. The motion of the electron in a plane perpendicular to $\vec{H}$ is more complicated and contains not one frequency Ω_0, but an entire set of frequencies $\omega_e = \Omega_0 - n\gamma$ (where n is an integer).

It is known that the energy absorbed by electrons in a constant electric field has a maximum when the natural frequency is zero, i.e., when

$$\Omega_0 = n\gamma. \tag{4}$$

In metals and semiconductors, this resonance becomes "smeared out" by collisions between the electrons and scatterers. Resonance is observed if the collision frequency ν is the smallest quantity

$$\nu << \Omega_0, \ \gamma. \tag{5}$$

Thus, if the conditions indicated above are satisfied, a resonance effect should be observed in the electron-hole plasma of a solid; we call it magnetic parametric resonance.

The time-averaged power Q absorbed per unit volume of the conductor equals

$$Q = \frac{1}{2} \operatorname{Re} \bar{\sigma}_{\mu\nu} E_\nu E_\mu^* , \tag{6}$$

where $\bar{\sigma}_{\mu\nu}$ is the time-averaged conductivity tensor of the electron gas. It can be easily obtained from Boltzmann's equation. At an arbitrary dispersion law, the tensor $\bar{\sigma}_{\mu\nu}$ is given by

$$\bar{\sigma}_{\mu\nu} = \frac{4\pi^2 e^2}{\gamma(2\pi\hbar)^3} \sum_{n=-\infty}^{\infty} \int_0^{\infty} d\epsilon \left(-\frac{\partial f_0}{\partial \epsilon}\right) \int_{p_z\,min}^{p_z\,max} dp_z\, m(\epsilon, p_z) v_{n\mu}^*(\epsilon, p_z) v_{n\nu}(\epsilon, p_z) \times$$

$$\times \frac{J_{i\lambda_n}(nq) J_{-i\lambda_n}(nq)}{\operatorname{sh} \pi\lambda_n} ; \tag{7}$$

where

$$\lambda_n = \frac{\nu + in\Omega_0}{\gamma}; \quad q = a\frac{\Omega_0}{\gamma}; \quad m(\epsilon, p_z) = \frac{1}{2\pi}\frac{\partial S}{\partial \epsilon}; \quad \tau = \int^t dt' \Omega(t');$$

$$v = \frac{\partial \epsilon}{\partial p}; \quad v_{n\mu}(\epsilon, p_z) = \theta^{-1} \int_0^{\theta} d\tau\, v_\mu(\epsilon, p_z, \tau) e^{-in\frac{\theta}{2\pi}\tau} ; \quad \theta = 2\pi \frac{m(\epsilon, p_z)}{m} .$$

$f_0(\varepsilon)$ is the equilibrium distribution, $S(\varepsilon, p_z)$ is the area of the intersection of the equal-energy surface $\varepsilon(\vec{p}) = \varepsilon$ with the plane p_z = const. We see thus that the resonance condition takes in the general case the form

$$p\Omega_0 = n\gamma. \tag{8}$$

In other words, there are p resonant series with n resonances in each. It is clear that the number of resonant series is exactly equal to the number of nonzero Fourier components of the electrons in a given direction. The resonance line shape depends essentially on the topology of the equal-energy surface. In the case of quadratic dispersion, the line shape and the character of the singularity can be easily obtained by using the results of [3].

We present the components of the time-averaged tensor $\bar{\sigma}_{\mu\nu}$ for a conductor with detenerate statistics in the case of isotropic and quadratic dispersion

$$\begin{aligned} \bar{\sigma}_{xx} &= \pi\sigma_0 \left[\frac{J_{i\lambda_1}(q) J_{-i\lambda_1}(q)}{\operatorname{sh}\pi\lambda_1} + \frac{J_{i\lambda_{-1}}(q) J_{-i\lambda_{-1}}(q)}{\operatorname{sh}\pi_{\lambda-1}} \right], \\ \bar{\sigma}_{xy} &= i\pi\sigma_0 \left[\frac{J_{i\lambda_1}(q) J_{-i\lambda_1}(q)}{\operatorname{sh}\pi\lambda_1} - \frac{J_{i\lambda_{-1}}(q) J_{-i\lambda_{-1}}(q)}{\operatorname{sh}\pi\lambda_{-1}} \right], \end{aligned} \tag{9}$$

$$\bar{\sigma}_{zz} = \sigma_0 \frac{\gamma}{2\nu},$$

$$\bar{\sigma}_{xz} = \bar{\sigma}_{yz} = 0, \quad \bar{\sigma}_{yy} = \bar{\sigma}_{xx}, \quad \bar{\sigma}_{yx} = -\bar{\sigma}_{xy}, \quad \sigma_0 = 2\frac{Ne^2}{m\gamma}.$$

If we use the known asymptotic forms of the Bessel functions [4], we obtain the resonance amplitude in various limiting cases.

1. Fundamental resonance (N = 1)

$$\bar{\sigma}_{xx} \sim \sigma_0 \frac{\gamma}{\nu} \begin{cases} a^2, & a \ll 1; & (10) \\ a^{-1}(1 + \sin a), & a \gg 1. & (11) \end{cases}$$

2. Resonance at higher harmonics (n >> 1)

$$\bar{\sigma}_{xx} \sim \sigma_0 \frac{\gamma}{\nu} \begin{cases} [n\alpha \,\mathrm{sh}\, a]^{-1} \exp[-2n\alpha(a \,\mathrm{ch}\, a - \mathrm{sh}\, a)], & (12) \\ \qquad \mathrm{ch}\, a = \alpha^{-1}, \quad n\alpha(a\,\mathrm{ch}\, a - \mathrm{sh}\, a) >> 1; & \\ n^{-2/3}, \quad n\alpha(a\,\mathrm{ch}\, a - \mathrm{sh}\, a) << 1, \quad \alpha = 1. & (13) \end{cases}$$

It is seen from these formulas that the amplitude of the resonance at high harmonics decreases slowly at $\alpha = 1$ with increasing n. At $\alpha \simeq 1$ (see (12) and (13) for the criterion) the amplitudes decrease exponentially with increasing n. For the fundamental resonance (n = 1), the case of greatest interest is (10), for it appears in forst order in α^{-1}, and the amplitude oscillates with changing depth of modulation α.

Insofar as we know, MPR has not yet been observed experimentally. The resonance can apparently be observed in bismuth in the frequency interval 10^9 sec^{-1} < γ < 10^{12} sec^{-1} at low temperatures and in magnetic fields on the order of a kilo-oersted.

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[1] I. M. Lifshitz, M. Ya. Azbel', and M. I. Kaganov, Electronnaya teoriya metallov (Electron Theory of Metals), Nauka, 1971.
[2] E. A. Kaner and V. M. Nabutovskii, Fiz. Tverd. Tela 4, 685 (1962) [Sov. Phys.-Solid State 4, 499 (1962)].
[3] E. A. Kaner, O. I. Lyubimov, I. E. Aronov, and R. I. Shekhter, Fiz. Tekh. Poluprov. 4, 57 (1970) [Sov. Phys.-Semicond. 4, 43 (1970)].
[4] E. T. Copson, Asymptotic Expansions, Cambridge Univ. Press.

EFFECT OF MAGNETIC BREAKDOWN ON NONLINEAR WAVE DAMPING IN METALS

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Nonlinear absorption of sound and helicons is calculated in the presence of magnetic breakdown, which consists of interband tunnel transitions of the conduction electrons in a strong magnetic field. It is shown that magnetic breakdown greatly extends the amplitude and wave-vector interval in which nonlinear effects are essential in wave damping. So strong an influence of magnetic breakdown is attributed to the singular random structure of the magnetic-breakdown spectrum.

1. It is known that acoustic and electromagnetic waves are damped in metals in the presence of strong spatial dispersion mainly as a result of their resonant interaction with a chosen group of conduction electrons. This process is usually treated within the framework of the linear theory, which neglects the effect of the wave field on the dynamics of the electron, and leads to a well known result, viz., Landau damping, which does not depend on the electron relaxation time t_0. The limits of applicability of the linear approximation are set by the inequality $t_0^{-1} >> \tilde{\omega}(\vec{q}, u_0)$, where $\tilde{\omega}$ is the characteristic frequency of the electron motion induced by the resonant interaction with the wave, $\vec{q}$ is the wave vector, and u_0 is the characteristic energy of electron-wave interaction.

In the usual (quasiclassical) situation, the condition t_0^{-1} is violated at such high values of q and u_0, that at first glance the observation of nonlinear effects in wave absorption by metals entails considerable experimental difficulties. The purpose in the present article is to demonstrate that magnetic breakdown in metals [1] changes qualitatively the entire picture of nonlinear wave damping (NWD), and makes it possible to observe MB at values of q and wave-field intensities I_0 perfectly attainable in contemporary low-temperature experiments.

2. We consider here closed magnetic-breakdown (MB) systems of electron orbits in p-space