2. Resonance at higher harmonics ($n >> 1$)

$$\bar{\sigma}_{xx} \sim \sigma_0 \frac{\gamma}{\nu} \begin{cases} [n\alpha\, \mathrm{sh}\, \alpha]^{-1} \exp[-2n\alpha(\alpha\, \mathrm{ch}\, \alpha - \mathrm{sh}\, \alpha)], & \\ \qquad \mathrm{ch}\, \alpha = \alpha^{-1}, \quad n\alpha(\alpha\, \mathrm{ch}\, \alpha - \mathrm{sh}\, \alpha) >> 1; & (12) \\ n^{-2/3}, \quad n\alpha(\alpha\, \mathrm{ch}\, \alpha - \mathrm{sh}\, \alpha) << 1, \quad \alpha \approx 1. & (13) \end{cases}$$

It is seen from these formulas that the amplitude of the resonance at high harmonics decreases slowly at $\alpha = 1$ with increasing n. At $\alpha \simeq 1$ (see (12) and (13) for the criterion) the amplitudes decrease exponentially with increasing n. For the fundamental resonance ($n = 1$), the case of greatest interest is (10), for it appears in forst order in α^{-1}, and the amplitude oscillates with changing depth of modulation α.

Insofar as we know, MPR has not yet been observed experimentally. The resonance can apparently be observed in bismuth in the frequency interval $10^9\ \mathrm{sec}^{-1} < \gamma < 10^{12}\ \mathrm{sec}^{-1}$ at low temperatures and in magnetic fields on the order of a kilo-oersted.

The author is deeply grateful to E. A. Kaner for a discussion of the results.

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EFFECT OF MAGNETIC BREAKDOWN ON NONLINEAR WAVE DAMPING IN METALS

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Submitted 25 September 1973
ZhETF Pis. Red. 18, No. 9, 587 — 590 (5 November 1973)

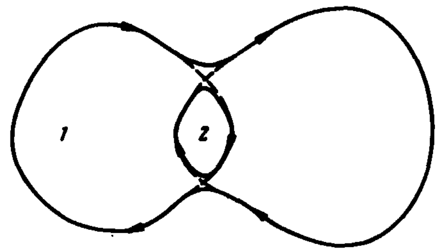
Nonlinear absorption of sound and helicons is calculated in the presence of magnetic breakdown, which consists of interband tunnel transitions of the conduction electrons in a strong magnetic field. It is shown that magnetic breakdown greatly extends the amplitude and wave-vector interval in which nonlinear effects are essential in wave damping. So strong an influence of magnetic breakdown is attributed to the singular random structure of the magnetic-breakdown spectrum.

1. It is known that acoustic and electromagnetic waves are damped in metals in the presence of strong spatial dispersion mainly as a result of their resonant interaction with a chosen group of conduction electrons. This process is usually treated within the framework of the linear theory, which neglects the effect of the wave field on the dynamics of the electron, and leads to a well known result, viz., Landau damping, which does not depend on the electron relaxation time t_0. The limits of applicability of the linear approximation are set by the inequality $t_0^{-1} >> \tilde{\omega}(\vec{q}, u_0)$, where $\tilde{\omega}$ is the characteristic frequency of the electron motion induced by the resonant interaction with the wave, $\vec{q}$ is the wave vector, and u_0 is the characteristic energy of electron-wave interaction.

In the usual (quasiclassical) situation, the condition t_0^{-1} is violated at such high values of q and u_0, that at first glance the observation of nonlinear effects in wave absorption by metals entails considerable experimental difficulties. The purpose in the present article is to demonstrate that magnetic breakdown in metals [1] changes qualitatively the entire picture of nonlinear wave damping (NWD), and makes it possible to observe MB at values of q and wave-field intensities I_0 perfectly attainable in contemporary low-temperature experiments.

2. We consider here closed magnetic-breakdown (MB) systems of electron orbits in p-space

(an example is shown in the figure). The electron MB energy spectrum corresponding to such configurations consists, as usual, of a set of discrete terms $E_n(p_z)$ (n is the number of term) that depend only on the projection p_z of the quasi-momentum $\vec{p}$ on the direction of the magnetic field $\vec{H}$ = (0, 0, H); ($\hbar^{-1}|E_{n+1} - E_n| \sim \Omega_0$; where Ω_0 is the characteristic Larmor frequency, and $|\partial E_n/\partial p_z| \sim v_0$, where v_0 is the characteristic electron velocity). Unlike the quasiclassical MB spectrum, the terms are randomly disposed, and $E_n(p_z)$ are random functions of p_z with a very small "quantum" scale of variation, $\Delta p \equiv \kappa p_0$ (p_0 is the characteristic momentum, $\kappa = \hbar\Omega_0/\varepsilon_F$ is the parameter of the quasiclassical approach, and ε_F is the Fermi energy), and with an equally small amplitude ($\sim \hbar\Omega_0$) of the random "trembling"[1]. If $\Omega_0 t_0 >> 1$, these singularities of the MB spectrum lead to NWD even at $u_0 << \hbar\Omega_0$. We shall henceforth deal with just this limiting quantum situation.



1 and 2 are the numbers of the energy bands, and the dashed lines delineate schematically the (small) MB region.

3. Under the influence of the effective field $u(z, t) \equiv u_0\cos(qz - \omega t)$ produced by a traveling plane wave (we assume for simplicity that $\vec{q} \,||\, \vec{H}$), the electron undergoes the transitions $|n, p_z\rangle \leftrightarrow |n', p_s\rangle$ ($|n, p_z\rangle$ is a stationary state with energy $E_n(p_z)$, n and n' are arbitrary, $p_s \equiv p_z + s\hbar q$, and $s = \pm 1, \pm 2, \ldots$). Since $u_0 << \hbar\Omega_0$, the only significant transitions are those connected by the resonance condition

$$\Big|\, |E_n(p_z) - E_{n'}(p_{s'})| - \hbar\omega \Big| \lesssim u_0 \text{ for all } |s'| \leqslant |s| \neq 0. \tag{1}$$

The structure of the resonant regions (1) in the presence of MB is quite unique. This particularly clearly pronounced if $qr_H \equiv \hbar q/\Delta p \gtrsim 1$ (r_H is the characteristic Larmor radius), when the randomness of $E_n(p_z)$ permits relation (1) to be satisfied in general only for $|s| = 1$, and consequently most resonant transitions take place independently of one another. The frequency of such two-level transitions is $\tilde{\omega} = u_0/\hbar$. This leads to the following (optimal) NWD criterion:

$$u_0/\hbar \gtrsim t_0^{-1} \tag{2}$$

If $qr_H << 1$ and $\omega << qv$, then the probabilities of transitions with $n \neq n'$ are negligibly small and (1) holds true only in the vicinity of the extremal points of $E_n(p_z)$. Using the relation $|\partial^2 E_n/\partial p_z^2| \sim (\kappa m_0)^{-1}$ (m_0 is the mass of the free electron), which is due to the sharpness of $E_n(p_z)$, it is easy to deduce from (1) that the "two-level" absorption and the criterion (2) hold true for all $\beta \equiv \hbar^2 q^2/\kappa m_0 u_0 >> 1$, and the smallest values of q that are compatible with (2) are of the order of $v^{-1}\sqrt{\Omega_0/t_0}$. Another situation arises if $\beta << 1$ and the probabilities of the transitions $|n, p_z\rangle \leftrightarrow |n, p_s\rangle$ are not small for all $|s| \lesssim \beta^{-1} >> 1$. Smallness of β means simultaneously that u_0 is much larger than the electron-energy uncertainty of localization at the wavelength q^{-1}. This enables us to describe the motion of the MB electron in terms of the classical Hamiltonian $\mathcal{H} = E_n(p_z) + u(z, t)$. This Hamiltonian corresponds to anomalously small effective masses $\sim \kappa m_0$, to anomalously large "classical" frequencies $\tilde{\omega} \sim q\sqrt{u_0/\kappa m_0}$, and the following MWD criterion:

$$q\sqrt{u_0/\kappa m_0} \equiv \sqrt{\beta}\, u_0 \hbar^{-1} \gtrsim t_0^{-1}; \quad (\beta << 1). \tag{3}$$

4. Let us estimate the possibility of experimentally observing NWD in the typical electron-impurity case $t_0 \sim 10^{-10}$ sec and $H \sim 10^4 - 10^5$ Oe. We find with the aid of (2) and (3) that under MB conditions we get nonlinear sound absorption ($u_0 \sim \varepsilon_F s^{-1}\sqrt{I_0/\rho_0 s_0}$, s_0 is the speed of sound, and ρ_0 is the density of the metal) at $I_0 \gtrsim 10^{-3}$ W/cm^2, and absorption of helicons ($u_0 \sim ev_0 H_1/cq$, e is the electron charge, c is the speed of light, and H_1 is the amplitude of the alternating magnetic field intensity) is obtained at $H_1 \gtrsim 10^{-3} - 10^{-2}$ Oe. To observe NWD with minimal I_0 (sound) and H_1 (helicons), we must have, respectively, $\omega \gtrsim 10^7 - 10^8$ sec^{-1} and $10^6 - 10^{-7}$ sec^{-1}. Comparison of (2, 3) with the criteria for quasiclassical NWD [3] shows that magnetic breakdown in metals decreases the values of I_0 and q needed for a noticeable manifestation of the nonlinear effects by a factor $\kappa^{-1} \sim 10^4 - 10^5$ and $\kappa^{-1/2} \sim 10^2$, respectively.

The quantum NWD effects are quite sensitive to deformation dislocation fields, which destroy the MB spectrum at dislocation densities $c_{dis} >> r_H^{-2}$ [2]. In the other limit, $c_{dis} << r_H^{-2}$, they give rise to oscillations of the energy $E_n(p_z)$, with amplitude $\delta\varepsilon \sim \hbar v_0 c_{dis}^{1/2}$ and frequency $\sim v_0 c_{dis}^{1/2}$. It is clear that at $\delta\varepsilon >> u_0$ the NWD effect can be neglected even when the electron-impurity t_0 satisfies relation (2). Estimates show that the influence of the dislocations on the NWD is appreciable if $c_{dis} \gtrsim 10^5$ cm^{-2}.

5. We present now a formula for the NWD coefficient (Γ) for the case when the "two-level" wave absorption ($\beta >> 1$) occurs under quasistatic conditions ($\omega t_0 << 1$):

$$\Gamma = (1 + (u_0 t_0)^2 \hbar^{-2})(1 + 4(u_0 t_0)^2 \hbar^{-2})^{-1/2} \Gamma_0(w). \tag{4}$$

Formula (4) was obtained in the t_0-approximation; a factor ~ 1 preceding $(u_0 t_0/\hbar)^2$ has been left out. Here $\Gamma_0(w)$ is the wave damping coefficient of the linear theory [4] and does not depend on t_0 of the MB, w(H) is the probability of interband tunneling (the MB probability), and $\Gamma_0(w) \sim \Gamma_0(0)$. In the linear limit $u_0 t_0 \hbar^{-1} \to 0$ we have $\Gamma \to \Gamma_0(w)$. If $u_0 t_0 >> \hbar$, then we have in accordance with (4) $\Gamma \sim u_0 t_0 \hbar^{-1} \Gamma_0 >> \Gamma_0$. The latter holds true only for the case $\beta >> 1$ and $\omega t_0 << 1$; in other situations ($\beta << 1$ or $\omega t_0 << 1$) we have $\Gamma \to 0$ as $u_0 \to \infty$. In the quasiclassical limit $w \to 0$ or 1, when the energy levels $E_n(p_z)$ are almost equidistant, independent two-level transitions (at $\omega t_0 << 1$) are no longer possible. Equation (4) is therefore valid only in the region $w(1 - w)\hbar\Omega_0 >> u$.

A detailed investigation of the entire situation at arbitrary ωt_0, and c_{dis} will be reported in a separate article.

It is my pleasant duty to thank I. M. Lifshitz and M. I. Kaganov for valuable discussions.

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1)The reason for this structure of $E_n(p_z)$ is that the orbits of different bands, coupled by the MB, have non-commensurable periods [2].

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LIMITING TEMPERATURE AND NONLOCALITY OF STRONG INTERACTION

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Submitted 26 September 1973
ZhETF Pis. Red. 18, No. 9, 590 — 593 (5 November 1973)

Arguments are presented in favor of the assumption than an exponential growth of the hadron density of states means nonlocality of the strong interaction. The dimensions of the corresponding nonlocality region are in strong disagreement with experiment.

1. The question of the existence of a limiting maximum permissible temperature T_0 of the order of the ion mass (see, e.g., [1])

$$T < T_0 \approx 130 - 160 \text{ MeV}. \tag{1}$$

has been extensively discussed of late. In addition to arguments based on the data on the interaction of high-energy particles1), Eq. (1) is favored by the statement that the hadron density of states increases exponentially,

$$\rho(E) = \int d\nu\, \delta(E - E_\nu) \sim \exp(E/T_0) \tag{2}$$

as the hadron mass $E \to \infty$. This statement is a consequence of the statistical bootstrap and of the dual resonance model. At low values of E, Eq. (2) agrees with the averaged empirical data.

Substitution of (2) into the expression for the partition function