one in a small space-time region. According to (6), the dimensions of this region are determined by the quantity (4).

5. The value of T_0 from (1) leads, when substituted in (4), to an elementary length $\ell \sim 10^{-13}$ cm. Yet experiments (particularly those aimed at verifying the dispersion relations) confirm even now the validity of the locality principle, all the way to scales on the order of 10^{-15} cm. There are grounds for assuming that ℓ is actually even much smaller and coincides with the quantum-gravitational length $\ell_g \sim 10^{-33}$ [7].

All the foregoing constitutes one more argument against the existence of a limiting temperature of the order of (1). It either does not exist at all, or its order of magnitude is much larger than the pion mass (for $\ell \sim \ell_g$, for example, we have $T_0 \sim 10^{19}$ GeV).[4] Accordingly, the hadron density of states either increases asymptotically more slowly than an exponential with linear argument, or is characterized by a temporal parameter that assumes much larger values as $E \to \infty$ than at small E.

We are grateful to E. L. Feinberg, who called our attention to the problem of the limiting temperature, for valuable discussions, and also to B. L. Voronov for a critical remark.

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1)Actually these arguments are not necessarily valid, since the indicated data admit also of another interpretation [2].

2)In particular, for a free nonrelativistic particle the quantity $G(\vec{x}, \vec{x}, t) \sim t^{-3/2}$ describes the "spreading" of an initially localized packet.

3)A more correct analysis based on the "smoothing" of the δ function in the right-hand side of the equation for the Green's function leads to practically the same result.

4)The idea of the limiting temperature in connection with quantum-gravitational effects is developed in [8].

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FEASIBILITY OF STRONG-COUPLING MAGNETIC CONDENSONS

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A mechanism whereby strong-coupling magnetic condensons are produced is indicated and systems that may be suitable for the realization of this mechanism are indicated.

Magnetic condensons (MC) are self-consistent states of the polaron type in homopolar crystals, and are produced by sufficiently strong magnetic fields, 10^5 Oe < H < 10^6 Oe [1]. Relatively simple quantitative results for the self-energy of the MC, for its effective mass, and for other characteristics can be obtained provided that the dimensionless parameter of the problem is

$$\gamma = \frac{\rho_0}{r_z} = \frac{1}{8\pi\rho_0}\frac{m^* D^2}{\hbar^2 \rho s^2} << 1 \qquad (1)$$

where $\rho_0 = \sqrt{c\hbar/eH}$ is the characteristic magnetic length, D is the deformation-potential constant, ρ is the density, s is the speed of sound, and m^* is the effective mass of the electron. For typical semiconductors, the inequality (1) is well satisfied up to fields 10^6 - 10^7 Oe, so that the electronic part of the MC wave function can be expanded in only the states of the lowest Landau band. A situation similar to that of condensons is quite common; thus, for example, in helium gas at densitites n_0 lower than a certain critical value $(n_0)_{cr} \simeq 2\times10^{21}$ cm^{-3}, a magnetic field of the order of 5×10^5 Oe is capable of producing the so-called large-radius ions (l.r.i.), which constitute regions of weak rarefaction of the gas, in which electrons are localized [2]. For the l.r.i. we have $r_z = (1/\pi n_0 a_0^2)(T/\mu H)$, where T is the temperature and a_0 is the length for electron scattering by the helium atom. The inequality (1) holds at $n_0 < 10^{21} cm^{-3}$ and $H < 5\times10^5$ Oe. At $\gamma \ll 1$, however, the self-energy turns out to be much lower than μH, so that either intermediate-coupling or weak-coupling MC are realized up to fields 10^6 Oe.

The purpose of the present article is to show that the self-energy of the MC and of the l.r.i. is close to μH for systems for which γ becomes of the order of unity in fields $10^5 - 10^6$ Oe, and the depth $|E_0^*|$ of the electronic level in the condenson well is much larger than the energy hs/ρ_0 of the phonons interacting most actively with the electrons, i.e., strong-coupling MC are possible in these systems in principle. Let us consider an electron in a classical non-polar elastic medium. The self-energy of the system, after minimization with respect to the components of the strain tensor, is a functional with respect to the electron wave function $\phi(r)$ [1]

$$F^*\{\phi\} = \int\left\{\frac{|(p + \frac{e}{c}A)\phi|^2}{2m^*} - \frac{D^2}{2\rho s^2}|\phi|^4\right\}dr - \mu H. \quad (2)$$

In the absence of a magnetic field, the dependence of F^* on the reciprocal radius k of the local state takes the form shown in Fig. 1 (curves a to c correspond to different forces binding the electron to the medium; $F^*(k)$ vanishes at $k = k_0 \simeq 15\hbar^2\rho s^2/m^*D^2$). In typical semiconductors, case a is realized as a rule, namely they either have k_0 on the order of the lattice constant so that the macroscopic description does not hold, or else an important role is assumed, even at $k < k_0$, by anharmonicities that are not accounted for in (2) and lead to a rapid growth of the free energy of the system (Fig. 1, dashed line). This result was obtained long ago by Deigen and Pekar [3]. In case c, the condenson can be produced without a magnetic field, but this case can hardly be observed in a solid. In helium gas, the case c corresponds to a density $n_0 > (n_0)_{cr}$, when the electron forms an ordinary negative ion [4].

We consider next systems close to the case b, in which stable condensons can still not be produced without a field, but the situation is close to the threshold. Such systems, possibly, are solidified inert gases, where the values of ρs^2 are much lower than for typical semiconductors, and D is apparently smaller, but not much. Measurements of the electron mobility have revealed in such substances no states of the condenson type, but the electron-phonon interaction turned out to be appreciable [5]. For helium, the case b corresponds to $n_0 \simeq (n_0)_{cr}$.

In case b, the magnetic field plays the role of the push that makes the system go over the threshold and can by the same token ensure the formation of strong-coupling condenson states. It is precisely in this case that a magnetic field of the order of $10^5 - 10^6$ Oe leads to $\gamma = 0$ in (1) and the expansion of $\phi(r)$ must take into account all the Landau bands; the functional (2) can be investigated in this case only by a direct variational method. In a magnetic field, the problem becomes axially symmetrical, and therefore F^* is determined by the values of two reciprocal radii, longitudinal ($k_{\parallel}$) and transverse ($k_{\perp}$); for concreteness, we choose the trial functions in the form $\phi(r) = (2\pi)^{3/2}k_{\perp}k_{\parallel}^{1/2}\exp[-k_{\perp}^2\rho^2 - k_{\parallel}^2z^2]$. The MC corresponds to a negative minimum of the surface $F^*(k_{\perp}, k_{\parallel})$, with $F^*(k_{\perp}^0, k_{\parallel}^0) \equiv F_0^*(\gamma)$. If $\gamma \ll 1$, then $k_{\perp}^0 = (2\rho_0)^{-1}$; the dependence of $F^*(k_{\perp}^0, k_{\parallel})$ on $k_{\parallel}$ is shown in Fig. 1 (curves d and e); the characteristic MC energies satisfy in this case the L:2:3:4: theorem [6]. At $\gamma > 0.3$, the quantity $F^*(\gamma)$ increases with increasing γ more rapidly than in the approximation of the lowest Landau band, i.e.,

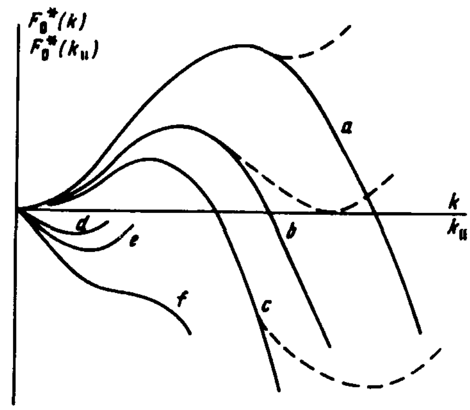

Fig. 1

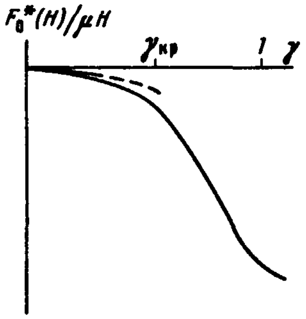

Fig. 2

inclusion of the remaining bands leads to a "strengthening" of the MC. Finally, when γ approaches the value $\gamma_{cr} = 3^{-1/4}\sqrt{\pi/6} \simeq 0.55$, the condenson minimum on the $F^*(k_\perp, k_\parallel)$ surface goes over to a saddle point; the situation corresponding to $\gamma \gtrsim \gamma_{cr}$ is shown somewhat arbitrarily by curve f of Fig. 1. Near γ_{cr}, the 1:2:3:4: theorem ceases to hold even approximately, and the conditions for the formation of a strong-coupling MC become much more favorable: at $\gamma = \gamma_{cr}$ the depth of the electronic level is $|E_0^*| \simeq 5|F_0^*|$. Further increase of γ does not lead to the appearance of a minimum on the $F^*(k_\perp, k_\parallel)$ surface. This behavior of the system is similar to the unlimited decrease of $F^*(k)$ of a harmonic lattice in the absence of a magnetic field (the "critical" MC is almost spherical in shape: $(k_\perp^0)_{cr} = (2/3)^{3/2}k_0$; $(k_\parallel^0)_{cr} = (2/3)^2 k_0$), so that at $\gamma > \gamma_{cr}$ the function $F^*(k_\perp, k_\parallel)$ acquires a minimum corresponding to the MC only when the anharmonicities are taken into account (the quantitative calcuations may call also for some modofication of the deformation-potential approximation). Since $(k_\perp^0)_{cr}$ and $(k_\parallel^0)_{cr}$ are each approximately half as large as k_0, the anharmonicities in cases close to b are still small at $\gamma = \gamma_{cr}$, so that the system is able to increase strongly the self-energy of the MC when γ exceeds γ_{cr}. The behavior of $F_0^*(\gamma)$ at $\gamma > \gamma_{cr}$ can be described quantitatively for helium gas with $n_0 \simeq 2\times10^{21}$ cm^{-3}, by using for $F^*\{\phi\}$ the expression given in [2]

$$F^*\{\phi\} = \int\left\{\frac{|(p + \frac{e}{c}A)\phi|^2}{2m} + n_0 T\left(1 - \exp\left[-\frac{2\pi\hbar^2 a_0}{mT}|\phi|^2\right]\right)\right\}dr - \frac{2\pi\hbar^2 a_0 n_0}{m} - \mu H. \quad (3)$$

At $\gamma \ll 1$ we can expand the exponential in (3) up to terms including $|\phi|^4$, and obtain as a result an expression similar to (2). The dependence of $F_0^*(\gamma)/\mu H$ on γ is shown in Fig. 2 by the solid line, while the dashed line is obtained in the approximation using the lowest Landau band. The value γ_{cr} is reached at $n_0 = 2\times10^{21}$ cm^{-3} and $H = 2.5\times10^5$ Oe. For $H = 10^6$ Oe, when $\gamma \simeq 1$, we have $|F_0^*| \simeq 0.5\times10^{-2}$ eV and $|E_0^*| \simeq 3.2\times10^{-2}$ eV. The local formation is then more like an ordinary ion than a large-radius ion. Figure 2 should describe adequately the behavior of MC in systems corresponding to the case b, if such systems can be found.

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