where E_0 - field amplitude, e and m - charge and mass of the electron, ω - frequency of light, and ν_{eff} (ε) - effective frequency of ion-ion collisions, we obtain $\varepsilon \sim 3 \times 10^4$ eV. The time between the ion-electron collisions is $\sim 3 \times 10^{-13}$ sec and the electron mean free path is $\sim 30\ \mu$. The volume of the heated plasma is consequently $\sim 3 \times 10^{-7}$ cm^3, giving an average energy $\sim 2 \times 10^3$ eV for the deuteron energy. This value agrees with the earlier estimate.

The investigations of plasma heating by powerful laser emission is continuing.

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QUANTUM MAGNETIC "TRAPS" IN METALS

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In investigations of low-temperature properties of metals placed in strong magnetic fields $\vec{H}$ it is customary to neglect the spatial inhomogeneity of the field. Under the conditions of the experiment the field satisfies under the experimental conditions the inequality R << L (R - characteristic Larmor radius, $L \sim H/|\nabla H|$). If the effects considered are non-vanishing in the zeroth approximation in the quasiclassical-approach parameter $\kappa = \hbar\Omega_0/\varepsilon_0 \sim 10^{-3} - 10^{-4}$ (Ω_0, ε_0 - characteristic Larmor frequency and energy, respectively), then the condition R << L actually makes it possible to assume, with good accuracy, that the field $\vec{H}$ is homogeneous. The situation is noticeably altered, however, when the quantum magnetic-breakdown effect (interband tunnel transitions) [1] becomes appreciable. We shall show in this note that under conditions of magnetic breakdown even very small inhomogeneities of $\vec{H}(\vec{r})$ greatly distort the electron energy spectrum and lead to the formation of unique quantum magnetic traps with characteristic dimensions $\sim\kappa L$. We shall carry out the analysis for a field $\vec{H} \equiv (0, 0, H_z(d))$, which arises, for example, in pulsed fields as a result of skin effect in the sample.

If the inhomogeneity of the magnetic field is sufficiently small, then it is convenient to investigate the dynamics of the conduction electron in terms of the energy spectrum that arises in a homogeneous magnetic field $\vec{H}_0 \parallel \vec{H}(x)$. We shall consider here the case when the magnetic breakdown in a field $\vec{H}_0$ ($\vec{A}_0 = (0, H_0 x, 0)$ perpendicular to one of the reciprocal-lattice vectors $\vec{b} = (0, b, 0)$ leads to the formation of a distinct band structure of the energy spectrum. For example, such a spectrum arises for the electron-orbit configuration shown in Fig. 1 (1, 2 - numbers of bands). The quasiclassical wave function of the electron can be represented here in the form

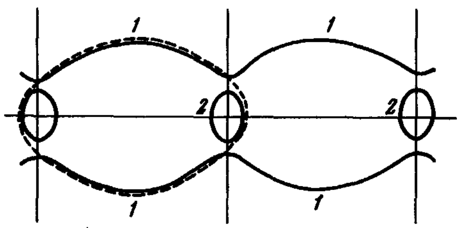

Fig. 1

$$E = E_n(\phi, p_z, H_0), \quad E_n(\phi) = E_n(\phi + 2\pi), \tag{1}$$

$$\psi^{(n)}_{\phi,p_z,p_{y_0}}(\mathbf{r}) = f^{(n)}_{\phi,p_z}\left(p_{y_0} + \frac{eH_0 x}{c}, \mathbf{r}, H_0\right) e^{i\frac{\phi x}{R_0} + i\frac{p_{y_0} y}{\hbar} + i\frac{p_{z_0} z}{\hbar}},$$

$$R = \frac{cb}{eH_0}, \tag{2}$$

$$f^{(n)}_{\phi,p_z}(p_y, \mathbf{r}, H_0) = f^{(n)}_{\phi,p_z}(p_y + b, \mathbf{r}, H_0); \quad f^{(n)}_{\phi,p_z}(p_y, \mathbf{r}) = f^{(n)}_{\phi,p_z}(p_y, \mathbf{r} + \mathbf{a});$$

$$\psi_\phi = \psi_{\phi + 2\pi}. \tag{2a}$$

Here n, ϕ, p_z, p_{y_0} - conserved quantum numbers, namely, p_z - projection of the quasimomentum on the vector $\vec{H}_0$, the discrete quantum number n is the number of the "magnetic" band, the continuous quantum number ϕ is the analog of the Bloch quasimomentum and numbers the states inside the "magnetic" band, and at the chosen gauge the degeneracy is with respect to the number p_{y_0}; $\vec{a}$ is the crystal-lattice vector, the function $f^{(n)}$ is analogous to the periodic multiplier of the "ordinary" Bloch function, and R_0 plays the role of the new period[1)] ($R \sim R_0$).

Using the results of [3], we shall point out several important properties of the functions E_n and $\psi^{(n)}$: (a) in states with quantum numbers n, ϕ, p_z, and p_{y_0} the average velocity transverse to the magnetic field differs from zero, viz., $v_x = (cb/e\hbar H)\partial E_n/\partial\phi \sim \varepsilon_0/b$, and $v_y = 0$, i.e., x is the direction in which the motion is infinite; (b) in the general case, when the magnetic-breakdown probability W is not small and is not too close to unity, the characteristic widths of the magnetic bands and the characteristic distance between them is of the order of $\hbar\Omega_0$; (c) the functions E_n and $\psi^{(n)}$ are extremely sensitive to changes of the parameter H_0, viz., the energy E_n of the magnetic band chantes by an amount $\sim\hbar\Omega_0$ when H_0 changes by an amount $\sim \kappa H_0$, and similarly $\partial\psi^{(n)}/\partial H_0 \sim \psi^{(n)}/\kappa H_0$.

[1)] The detailed structure of $f^{(n)}$, which can be readily determined in the quasiclassical approximation with the aid of the method of [3], is immaterial here and will not be written out in what follows.

If $\nabla H \neq 0$, then the degree of the influence of the inhomogeneity of $H(x)$ on the dynamics of the process is determined, in accordance with the foregoing estimates, by the relation $\alpha = \delta H/\kappa H = R/\kappa L$, where $\delta H \equiv R\nabla H$ is the change of the magnetic field over the period of R_0. We shall assume henceforth that $\alpha \ll 1$. It can be shown that this inequality is compatible with the relation $i[\hat{\phi}, \hat{x}] = cb/eH(\hat{x}) \sim R \ll \kappa L$ makes it possible (in the first approximation in α) to regard the quantum number n as conserved (p_z and p_{y_0} are rigorously conserved) and to describe the motion of the electron in a weakly inhomogeneous field with the aid of the classical Hamiltonian

$$\hat{\mathcal{H}}^{(n)}_{cl}(p_x, x) \equiv E_n(\phi(p_x), p_z x), \quad \phi(p_x) \equiv c b p_x / e\hbar H(x), \tag{4}$$

i.e., $\hat{\phi}$ and $\hat{x}$ commute in this approximation.

The motion of the particle in the phase space (ϕ, x) follows the trajectory $x = x_n(\phi, E, p_z)$, where the function $x_n(\phi)$ is determined by the conservation law $E_n(\phi, p_z, H(x)) = E$. The function $x_n(\phi)$, just as $E_n(\phi)$, is periodic, and consequently motion with a given n is finite. The width of the produced quantum "trap," as follows from the estimates above, is of the order of κL. This is illustrated by the scheme of inclined bands $E_n(x)$ (ϕ is fixed (Fig. 2); the shaded bands represent the allowed energy values corresponding to different values of ϕ). As seen from Fig. 2, a given value of E generates a set of "traps" corresponding to different magnetic-band numbers. Quantum tunnel transitions take place between regions of finite motion having different values of n. The probabilities of the transitions are $\sim \exp(-d/R) \ll 1$, where $d \sim \kappa L$ is the minimum value of $|x_{n+1}(\phi) - x_n(\phi)|^2$.

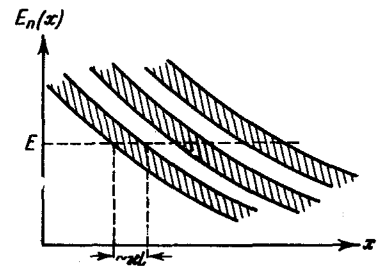

Fig. 2

When $\kappa L \ll \ell$ (the mean free path), the "traps" should strongly affect the macroscopic properties of the metal; in particular, they should change its transverse electric conductivity σ_{xx}. In a homogeneous magnetic field, the motion along the x axis is infinite (see above), yielding the estimate $\sigma_{xx} \sim \sigma_0$ (σ_0 - electric conductivity when $H = 0$ [3, 5]. In a weakly inhomogeneous field, the electron executes in the "trap" a periodic motion with a characteristic frequency $\tilde{\Omega} \sim \alpha\Omega_0$. If $\Omega t_0 \gg 1$ (t_0 - relaxation time), we have in accordance with the general theory of galvanomagnetic phenomena [6] $\sigma_{xx} \sim \sigma_0/(\tilde{\Omega}t_0)^2 \sim (\kappa L/\ell)^2\sigma_0 \ll \sigma_0$. These estimates can also be obtained directly from the exact expressions for the current j_x and for the non-equilibrium addition to the density matrix $\rho_n(p_x, x, p_z)$, which satisfies the classical kinetic equation

$$\begin{gathered}\{\mathcal{H}^{(n)}_{cl}(p_x, p_z, x), \rho_n(p_x, p_z, x)\}_{p_x x} + \rho_n/t_0 = e\mathcal{E}\frac{\partial f_0}{\partial E} v_x(\phi(p_x), x, p_z), \\ j_x = \frac{eH}{(2\pi\hbar)^3 c}\sum_n \int dp_z \int_0^{2\pi} d\phi\, \rho_n(\phi, p_z, x) v_n(\phi, p_z, x).\end{gathered} \tag{5}$$

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[2] The latter phenomenon is analogous to the Zener breakdown [4].

Here $\{\ldots\}_{p_x x}$ are the classical Poisson brackets, $\mathcal{E}$ the electric field intensity, and f_0 the Fermi function. We present without calculations the expression for $j(x)$ obtained for the configuration of Fig. 1 under the assumption that $W \approx 1$ (but $(1 - W)\kappa L \gg R$), when the band spectrum (1) represents slightly broadened Landau levels corresponding to the trajectories shown dashed in Fig. 1:

$$j_{cl}(x) = \frac{e^4 \mathcal{E}}{(2\pi\hbar)^3} \frac{4(1-W)}{f_0} \left(\frac{eH\hbar}{c}\right)^2 \left(\frac{H}{|\nabla H|}\right)^2 \int dp_z \frac{\partial S}{\partial \zeta} \Big/ S^2(\zeta, p_z).$$

The integration is over all the p_z for which magnetic breakdown takes place, and $S(\zeta p_z)$ is the area enclosed by the dashed trajectory in Fig. 1 (ζ is the Fermi energy).

The inequalities $R \ll \kappa L \ll \ell$, which are necessary for the occurrence of the "traps," can be readily satisfied in fields of 10^5 Oe if L lies between 1 and 10 cm. When $R \gtrsim \kappa L$, the tunnel transitions between the "traps" become appreciable, and the character of motion is greatly changed. These questions will be dealt with in a separate article.

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EXACT SOLUTION OF THE DECORATED ISING MODEL WITH NONMAGNETIC IMPURITIES

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The free energy, and hence all the remaining thermodynamic quantities, are known at present for only one system that undergoes a phase transition and has a finite interaction radius, namely the two-dimensional Ising lattice with nearest-neighbor interaction. This problem was solved by Onsager [1]. Great interest attaches therefore to an investigation of the phase transition in other models that admit of an exact solution. We obtain in this paper an exact solution of the decorated plane Ising model with nonmagnetic impurities. We define as decorated the lattice shown in Fig. 1 and consisting of two sublattices. The light and dark circles represent different atoms, A and B respectively, with magnetic moments that can be directed either upward or downward ($\sigma = \pm 1$). Only the neighbors joined by solid lines interact. The Hamiltonian of this system has the usual form for the Ising model[1)]:

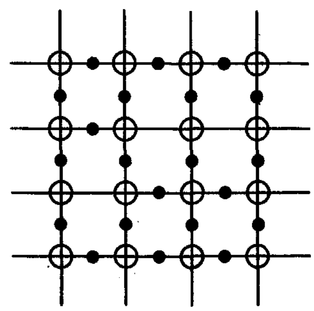

Fig. 1

[1)] A similar model (without impurities) was investigated in detail by Syozi and Nakano [2].