c) The spontaneous magnetization below the transition point behaves like $[\beta(c)\Lambda(\alpha\tau)]^{1/8}$. Accurate to a factor on the order of unity, we have

$$\beta(c) = \exp\frac{(1+\sqrt{2})^2\pi}{8c}\left(1 - 2c\frac{3+\sqrt{2}}{\pi(1+\sqrt{2})^2}\right) \tag{25}$$

We note that the lattice magnetization due to the A-spins does not reach saturation at T = 0, since a fraction of the A-atoms is isolated from the remainder of the system as a result of the nonmagnetic impurities.

In conclusion, we can express the hopes that the character of the singularities obtained in this paper, as well as the (qualitative) dependence of the transition temperature on the impurity concentration, are possessed not only by the model under consideration, but also by a broader class of systems, although the symmetrical form of the specific-heat singularity is a typical property of this model, connected with the Kramers-Wannier symmetry of the initial impurity-free model.

[1] L. Onsager, Phys. Rev. 65, 117 (1944).
[2] I. Syozi and H. Nakano, Progr. Theor. Phys. 13, 69 (1955).
[3] L. D. Landau and E. M. Lifshitz, Statisticheskaya Fizika, 1964 [Statistical Physics, Addison-Wesley, 1958].
[4] B. Kaufman and L. Onsager, Phys. Rev. 76, 1244 (1949).
[5] C. N. Yang, Phys. Rev. 85, 808 (1952).
[6] M. Fisher, Physica 25, 521 (1959).

POSSIBILITY OF OBSERVING ZERO SOUND IN NUCLEI BY RADIATIVE PION CAPTURE

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It is well known [1, 2] that there are four possible types of volume collective excitations of nulcear matter (NM), which can be regarded, in accord with the Landau theory of the Fermi liquid, as different types of zero sound in NM: density waves, spin waves, isospin waves, and coupled spin-isospin waves (henceforth denoted by the indices 0, s, i, and si). In this paper we determine the information that can be obtained concerning these excitations by investigating the interaction between slow pions and nuclei.

The exceptional properties of slow pions (with energy up to several MeV) are connected with their small scattering length compared with the radius of action of the nuclear forces and with the average distance between the nucleons of the nucleus (Ericson [3]), as a result of which the interaction between a slow pion and a nucleus can be described (just like the interaction between a slow neutron and a molecule or a crystal) with the aid of a sum of Fermi pseudopotentials. If we confine ourselves to a consideration of the interaction in the s-state, then the effective Hamiltonians of the π^--meson radiative capture and of the scattering of the pion by the nucleus are [4, 5]

$$H^{(r)} = 4\pi i \sum_{\ell} A_\sigma \tau^- (\vec{\epsilon}\vec{\sigma}_\ell)\delta(r - r_\ell), \tag{1}$$

$$H^{(s)} = 4\pi \sum_{\ell}(B_0 + B_1\vec{\tau}_\pi\vec{\tau}_\ell)\delta(r - r_\ell), \tag{2}$$

where $\vec{r}$ and $\vec{r}_\ell$ are the radius vectors and $\vec{\tau}_\pi$ and $\vec{\tau}_\ell$ are the isospins of the pion and of the ℓ-th nucleon, $\vec{\sigma}_\ell$ is the nucleon spin, $\vec{\varepsilon}$ is the photon polarization vector, $A_0 = 3.4 \times 10^{-2} m_\pi^{-2}$, and $B_{0,1} \sim 0.1 m_\pi^{-2}$ (m_π is the pion mass; $\hbar = c = 1$).

The cross sections for the different interactions between pions and nuclei are expressed in terms of the NM correlators. We define the latter by starting from the Fermi-liquid model of NM [6], in which the state of NM is described by the quasiparticle distribution function over the coordinates and momenta, $n(\vec{p}, \vec{r}, t)$, which is simultaneously the density matrix with respect to the spin and isospin variables. It is assumed here that the function F, which enters in the Landau kinetic equation [7] for n and which characterizes the interaction between the quasiparticles, is of the form

$$F = F^{\circ} + 4\vec{\sigma}\vec{\sigma}' F^{(s)} + 4\vec{\tau}\vec{\tau}' F^{(i)} + 16(\vec{\sigma}\vec{\sigma}')(\vec{\tau}\vec{\tau}')F^{(si)}, \tag{3}$$

where $F^{(\dot{\alpha})}$ are scalar functions of $\vec{p}$ and $\vec{p}'$. In accordance with such a structure of the function F, we can determine the four correlators of NM

$$\Phi^{(\dot{\alpha})}(q, \omega) = \frac{C_\alpha}{2\omega} \mathrm{Sp}\,\Gamma^{(\dot{\alpha})}\Gamma'^{(\alpha)} \int dr\, dt\, e^{-iqr + i\omega t} \langle \rho(r_1, t_1)\rho(r_2, t_2) \rangle, \tag{4}$$

where

$$\Gamma^{(\dot{\alpha})} \equiv \{1, \sigma_l, \tau_l, \sigma_l, \tau_l\}; \quad \rho(r, t) = \int n(p, rt) \frac{d^3p}{(2\pi)^3};$$

$$r = r_1 - r_2, \quad t = t_1 - t_2; \quad C_\alpha \equiv \{1, \frac{4}{3}, \frac{4}{3}, \frac{16}{9}\}$$

(<...> denotes quantum-mechanical averaging).

If existence of α-sound is possible in NM (α = 0, s, i, si), then the correlator $\Phi^{(\dot{\alpha})}$ has the following structure[1)]

$$\Phi^{(\alpha)}(q, \omega) = \Phi_c^{(\dot{\alpha})}(q, \omega) + \Phi_d^{(\dot{\alpha})}(q, \omega),$$
$$\Phi_c^{(\dot{\alpha})}(q, \omega) = 2\pi^{-3} p_0 m^* R_c^{(\dot{\alpha})} \omega \gamma^{(\dot{\alpha})} \{(\omega - s^{(\dot{\alpha})} q)^2 + \gamma^{(\dot{\alpha})2}\}^{-1}, \tag{5}$$
$$(\omega > 0)$$
$$\Phi_d^{(\dot{\alpha})}(q, \omega) = \frac{\omega}{qv_0} \pi^{-2} p_0 m^* R_d^{(\dot{\alpha})}\, \theta(qv_0 - \omega),$$

where $s^{(\dot{\alpha})}$ is the velocity and $\gamma^{(\dot{\alpha})}$ is the damping of the α-sound, $R_c^{(\dot{\alpha})}$ is a constant on the order of unity, $R_d^{(\dot{\alpha})} \equiv R_d^{(\dot{\alpha})}(\omega/qv_0)$ 1, $\theta(x) = (1 + \mathrm{sgn}\, x)/2$, and p_0 and v_0 are the limiting momentum and velocity of the nucleon distribution, with $m^* = p_0/v_0$ (if the α-sound can not propagate, there is no pole term of $\Phi_c^{(\dot{\alpha})}$).

Formula (5) corresponds to a breakdown of the cross sections of the different processes into two terms, $d\sigma = d\sigma_{(c)} + d\sigma_{(d)}$, where the smooth term $d\sigma_{(d)} \sim \Phi_d$ describes the direct process, i.e., the interaction of the pion with individual nucleons of the nucleus (with allowance for the correlation between the nucleons, and $d\sigma_{(c)} \sim \Phi_c$ describes a process accompanied by excitation of the collective excitations of the NM. In particular, such a

[1)]Expressions (5) generalize the well known expressions for correlators in a Fermi liquid [8] to the case of nonzero $F^{(i)}$ and $F^{(si)}$.

structure should be possessed by the cross section for radiative pion capture ($\pi \to \gamma$), which is expressed, according to (1), in terms of the correlator $\Phi^{(si)}$

$$d\sigma_{\pi\to\gamma} = (2v\rho_0)^{-1} |A_0|^2 \Phi^{(si)}(q,\omega)\, d^3k' \equiv d\sigma_{(c)\pi\to\gamma} + d\sigma_{(d)\pi\to\gamma}, \tag{6}$$

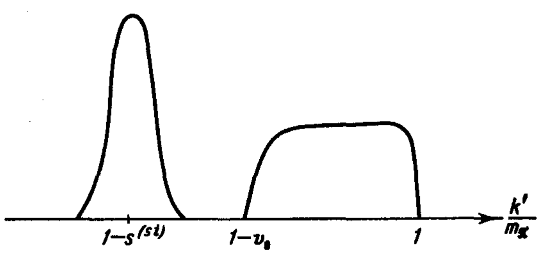
where $\vec{q} = \vec{k} - \vec{k}'$, $\omega = \varepsilon - k'$, ε and $\vec{k}$ are the energy and momentum of the incident pion, $\vec{k}'$ is the momentum of the radiated photon, $v = k/m_\pi$, and ρ_0 is the density of the NM. The pole term in the cross section of this process describes the radiative capture of the pions with excitation of si-sound, and it can be shown that in this case

$$\frac{\sigma_{(c)}}{\sigma_{(d)}} \sim \frac{1 - \frac{v_0}{s}}{\left|\ln\left(1 - \frac{v_0}{s}\right)\right|} \quad (s \equiv s^{(si)}). \tag{7}$$

In radiative capture of pions with excitation of si-sound, the following conservation laws are satisfied:

$$m_\pi + \frac{k^2}{2m_\pi} = k' + s^{(si)}|\mathbf{k} - \mathbf{k}'|. \tag{8}$$

As a result, a sharp maximum should be observed in the spectrum of the emitted photons at $k' \approx m_\pi(1 - s^{(si)})$. (The function $d\sigma_{\pi\to\gamma}/dk'(m_\pi - k')^{-1}$ is shown schematically in the figure). The ratio of the width of this maximum to the width of the plateau corresponding to the direct process $\pi \to \gamma$ is $\Gamma_c/\Gamma_d \sim (m_\pi/Mv_0)^2$, where M is the nucleon mass. Experimental observation of such a maximum could serve as a check on the existence of si-sound in nuclei.



Using (2), we can determine the cross sections for the scattering of slow pions by nuclei ($\pi \to \pi$). They are expressed in terms of the correlators $\Phi^{(0)}$ and $\Phi^{(i)}$ and contain pole terms corresponding to excitation of 0- and i-sound. The cross section ratio $\sigma_{(c)\pi\to\pi}/\sigma_{(d)\pi\to\pi}$ are determined as before by formula (7) (with $s = s^{(0)}$ or $s = s^{(i)}_0$: the ratio of the widths of the corresponding pion distributions is $\Gamma_c/\Gamma_d \sim (m_\pi v/Mv_0)^2$.

We note that interaction in the p-state comes into play when the pion energy increases. Then i-sound is excited in the $\pi \to \gamma$ reaction besides the si-sound, and s- and si-sound is excited in the $\pi \to \pi$ reaction besides 0- and i-sound.

[1] A. E. Glassgold, W. Heckrotte, and K. H. Watson, Ann. of Phys. 6, 1 (1959).
[2] S. Hatano, Progr. Theor. Phys. 24, 418 (1960).
[3] T. E. O. Ericson, Pion Interaction with Nuclei, Preprint TH. 716, 1966.
[4] M. Ericson and T. E. O. Ericson, Ann. of Phys. 36, 323 (1966).
[5] J. Delorme and T. E. O. Ericson, Phys. Lett. 21, 98 (1966).
[6] A. B. Migdal, Zh. Eksp. Teor. Fiz. 43, 1940 (1962) [Sov. Phys.-JETP 16, 1366 (1963)].
[7] L. D. Landau, ibid. 30, 1058 (1956) [3, 920 (1956)].
[8] A. I. Akhiezer, I. A. Akhiezer, and I. Ya. Pomeranchuk, ibid. 41, 478 (1961) [14, 343 (1962)]; Nucl. Phys. 40, 139 (1963).