A more detailed description of the setup and of the results obtained with it is being readied for publication.

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HIGHLY EXCITED STATES OF ATOMIC NUCLEI

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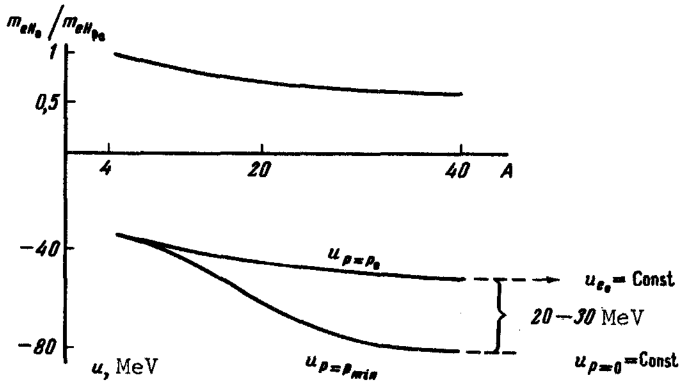
Submitted 21 May 1968
ZhETF Pis. Red. 8, No. 2, 86 - 89 (20 July 1968)

Recent experiments on inelastic scattering of electrons [1] and protons [2] by nuclei have revealed highly-excited resonant states of the nuclei. An attempt was made to interpret the results of these experiments within the framework of the usual shell model [1 - 3], it being assumed that the indicated resonance correspond to excitation of the internal shells. It turned out here, however, that the energies of the $^1S_{1/2}$, $^1P_{1/2}$, and $^1P_{3/2}$ levels increase with increasing atomic weight, reaching in the case of $^1S_{1/2}$ the values 40 MeV for A = 16 and 60 - 80 MeV for A = 40 [1, 2]. This contradicts the usual shell model, according to which the depth of the potential well is of the order of 45 - 55 MeV, and concequently the energy of any level should be smaller than this quantity.

The purpose of the present paper is to describe a possible resolution of this contradiction.

It is appropriate to recall that, according to modern many-body theory, a so-called single-quasiparticle branch of the spectrum exists in the vicinity of the Fermi surface for a system of interacting Fermi particles. The excitations (quasiparticles) behave like a system of non-interacting particles moving in a certain average self-consistent field. In general, this field is nonlocal.

According to this point of view, the shell model describes a quasi-single-particle excitation spectrum of the system. Owing to the strong interaction between the nucleons of the nucleus, there are no sufficiently convincing calculations of the parameters of the self-consistent field on the basis of the forces acting between the free nucleons. These parameters are usually chosen such as to make the calculated spectrum of the low-energy excitations (i.e., the excitations at the Fermi surface) coincide with the experimentally observed ones. It turns out that the depth of the potential well must be assumed, at least for medium and heavy nuclei, to equal 50 - 55 MeV, and its radius to equal approximately the nuclear radius $R = r_0A^{1/3}$; the nonlocality can be neglected. Thus, by definition, the assumed potential well should describe only the spectrum of the low-lying excitations, and not at all the positions of the deep levels of the nucleus far away from the Fermi surface. In order to find the positions of the deep levels, it is ncessary to determine the quasi-single-particle potential not at the Fermi surface, but far from it. In this case, likewise, it is hardly possible to calculate the parameters of the potential with the required degree of reliability. The considerations that follow are therefore only qualitative and tentative.



The deepest level corresponds to the smallest possible momentum in the bounded system, i.e., $p_{min} \sim 1/R$. Even for $A > 20$, the momentum p_{min} is much smaller than p_0, the Fermi momentum connected with the average density ρ of the nuclear matter by the relation $\rho = 2p_0^3/3\pi^2$ (we are using a system of units in which $h = m_{nucl} = 1$ and the energy is in MeV). We shall therefore assume henceforth that the momentum of the nucleon at the lowest level is zero. To ascertain how the self-consistent potential varies when the momentum changes from $p = p_0$ to $p = 0$, we consider first the Hartree-Fock approximation. The contribution of the Hartree term does not depend on the momentum. The dependence of the exchange term on the momentum is determined by the interaction radius. If the radius of the forces is noticeably larger than the mean distance between particles, then the contribution of the Fock term in the vicinity of the Fermi surface is half as large as at $p = 0$. For forces with an action radius that is small compared with the average distance, the contributions at $p \approx p_0$ and $p = 0$ are close to each other. We use a two-parameter model of nuclear forces, according to which the interaction between nucleons represents attraction forces (with allowance for exchange) at large distances and infinitely strong repulsion at small distances [4]1). An elementary calculation shows that the potential well at $p = 0$ is deeper by 15 - 20 MeV than at $p = p_0$. Another interesting feature is that the effective mass characterizing the dependence of the potential on the momentum differs greatly at $p = p_0$ and $p = 0$. Most investigators agree, on the basis of an analysis of the optical potential of the nuclei and the spectrum of the low-energy excitations, that m_{eff} is close to unity at $p = p_0$. This result can be understood as meaning that the influence of the large-radius forces, which tends to decrease m_{eff} compared with unity, is almost completely offset by the influence of the short-range repulsion. No such cancellation occurs at $p \approx p_0$, since the contribution made to m_{eff} by the short-range repulsion is much smaller than at $p \approx p_0$, and larger for the long-range attraction. In the indicated model, m_{eff} turns out to equal 0.5 at $p \approx 0$.

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1) The latter is replaced by a pseudopotential.

The qualitative considerations presented above are confirmed also by a much more complicated calculation in the Brueckner approximation [5]. It turns out that the potential well at $p = 0$ is deeper by 25 - 30 MeV than at $p = p_0$, with m_{eff} equal to 0.75 at p_0 and 0.5 at 0. Unfortunately, the applicability of Brueckner's approximation to nuclear matter has not been convincingly demonstrated, so that the figures presented can be regarded only as a manifestation of a tendency, namely that $|U_{p_0}| < |U_0|$ and $m_{eff_p} > m_{eff_0}$.

It is doubtful whether any other approximation would lead to a very reliable calculation of $U_{p=0}$. It is apparently most desirable to determine the parameters of the potential phenomenologically, on the basis of the experimentally-determined position of the levels.

The saturation of the nuclear forces, which is manifest in the constancy of both the average energy per nucleon and the internal density, takes place for nuclei with $A > 16 - 20$ [6]. Starting with approximately the same values of A, the depth of the potential well both at the Fermi surface and for $p = 0$. For sufficiently large A, therefore, the energy of the $^1S_{1/2}$ level should be given by a formula of the type

$$E_{1s} = |U_0| - \frac{p^2_{min}}{2m_{eff_0}}, \tag{1}$$

where $p_{min} \sim 1/R \sim A^{-1/3}$ and m_{eff_0} is the effective mass at $p = 0$.

Thus, for medium and heavy nuclei E_{1s} depends little on A, inasmuch as $|U_0| \gg p_{min}/2m_{eff_0}$. At the same time, in the case of the lightest nuclei the characteristic momenta of the nucleons at the Fermi surface and at depth are close to each other, so that for these nuclei the potential well of the outer and inner nucleons is the same, and m_{eff} is close to unity. The figure shows qualitatively the dependence of m_{eff} and of $U_{0_{min}}$ on A. A certain increase in $|U|_{p=p_0}$ is connected with the fact that the density of the light nuclei is somewhat lower than that of the heavy and medium ones.

A presently prevalent opinion, particularly among experimenters, is that the concept of the shell model in terms of quasiparticles presents essentially no new results. The very difference between quasiparticles and particles, especially since m_{eff} is close to unity at p_0, is in many respects only terminological. If the proposed explanation of the data of the experiments in [1,2] were correct, this would serve as a very convincing confirmation of the validity of the quasiparticle approach to the shell model.

In conclusion, I thank L. A. Sliv and V. N. Efimov. I am particularly grateful to Leksin, whose lecture stimulated the writing of the present paper.

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