the solution of the ordinary dynamic problem, and $E_1'(\vec{r})$ is given by the convolution

$$E_1'(\mathbf{r}) = -i\left\{\left(\frac{\partial}{\partial z} + \frac{\partial}{\partial x}\right) E_1^0\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial x}\right)(\mathbf{k}_1 \mathbf{u})\right\} * G(x, z). \qquad (10)$$

As expected, unlike the well-known case of diffraction electron-microscope images, the connection between the displacement field $\vec{u}(\vec{r})$ and its x-ray image $E_1'(\vec{r})$ is nonlocal. It is not surprising that different attempts at a local interpretation of the x-ray images of distortion fields were unsuccessful.

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NEUTRINOS IN ANISOTROPIC COSMOLOGICAL SOLUTIONS

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The singular behavior of the weakly-interacting neutrinos in anisotropic and homogeneous cosmological models and the appreciable influence of these particles on the dynamics of the models themselves were noted in [1, 2].

The hypothesis was advanced that the evolution is naturally subdivided into the following stages: 1) practically total equilibrium of all particles, including the neutrinos, and 2) the stage of free neutrinos, the momentum of which increases by virtue of the "blue shift" produced by compression along one of the axes.

Misner [3, 4] proposes that the equilibrium period is followed by a stage in which the neutrino influence can be described by using the concept of viscosity.

For this purpose it is obviously necessary that the neutrino mean free path not be too large; the characteristic length is ct, where the time t is reckoned from the singularity. Misner arrives at the conclusion that the viscosity, causing heating (entropy growth) of the matter, eliminates the second stage of free neutrinos in the strongly anisotropic expansion phase. According to Misner, the neutrinos become free when the expansion anisotropy is no longer large.

The two different points of view, Misner's (M) and ours (O), lead to different laws for the time variations of such quantities as the temperature (M: $T \sim t^{-1/5}$, O - $T \sim t^{-5/18}$), the entropy (M: $S \sim t^{2/5}$, O: $S \sim t^{1/6}$), and particularly the energy of one neutrino (M: $E_1 \sim T \sim t^{-1/5}$, O: $E_1 \sim t^{1/9}$. The total energy density of the neutrinos is proportional in both cases to the energy density of the strongly-interacting particles

$$\mathrm{M:}\ \nu E_1 \sim t^{-3/5} t^{-1/5} \sim t^{-4/5} \sim T^4;\ \mathrm{O:}\ \nu E_1 \sim t^{-11/9} t^{1/9} \sim t^{-10/9} \sim T^4.$$

The exponents are given throughout for the simplest case of axisymmetric motion and for a quadratic dependence of the cross section of the neutrino processes on the energy, as well as for the period when the matter, including the neutrinos, still does not influence the dynamics of the expansion.

Thus, the greatest difference in the conclusions concerns the behavior of the neutrinos: according to Misner, the ratio of the average momentum of the neutrinos traveling in different directions to the momentum of the other particles remains constant. In the model defended in this communication, the ratio of the momentum of the neutrinos moving along one of the axes to the momentum along the other axes and to the momentum of the other particles continues to increase until isotropization of the expansion sets in. This difference between the conclusions calls for a more detailed examination of the question.

Solving the equation for the energy

$$\frac{d\epsilon}{dt} + \frac{4\epsilon}{3t} = \frac{4\eta}{3t^2} = \frac{4}{9}\frac{\epsilon_\nu \tau}{t^2} = \frac{4}{9}\frac{k\tau\epsilon}{t^2},$$

we can readily show that the following relation holds true for the total energy density ε and for the path time τ

$$\epsilon = \epsilon_a \left\{ 1 + \frac{k(2m+3)}{6m} \left[\frac{\tau}{t} \left(\frac{1 - \frac{k(2m+3)}{6m}\frac{\tau_0}{t_0}}{1 - \frac{k(2m+3)}{6m}\frac{\tau}{t}} \right) - \frac{\tau_0}{t_0} \right] \right\}^{4/(2m+3)}$$

where η is the viscosity, 2m the exponent in the energy dependence of the interaction cross section $\sigma \sim E^{2m} \sim T^{2m}$, $\tau = \tau_0(\varepsilon_0/\varepsilon)^{(2m+3)/4}$ is the free path time, $k = \varepsilon_\nu/\varepsilon$ at equilibrium[1)] (we assume that $k = \varepsilon_\nu/\varepsilon = \mathrm{const}$), the index "0" denotes quantities at the instant when the viscosity is "turned on," ε_a is the energy density that would obtain were it to vary adiabatically also after t_0, $\varepsilon_a = \varepsilon_0(t/t_0)^{-4/3}$. It is clearly seen from (1) that the Misner regime, in which $\varepsilon \gg \varepsilon_a$, sets in when the denominator in the round brackets tends to zero:

[1)] When using the viscosity concept we must assume that k does not differ from the equilibrium value. The value of the factor k is obtained in the following manner: For $T \approx 0.5$ GeV, taking e, μ, ν_e, ν_μ, γ, and π into account, we find that k equals approximately 1/4. For $T > 3$ GeV we have in equilibrium e, μ, ν_e, ν_μ, baryons, their antiparticles, photons, and scalar and vector mesons. Whether all these particles can be regarded as independent at this temperature is unknown. If we regard as independent e, μ, ν_e, ν_μ, γ, and the baryon octet, then $k = \varepsilon_{\nu_\mu \nu_e}/\varepsilon = 7/81 \approx 1/12$. If we regard also as independent the baryon decuplet, the octet of scalar meson, and 9 vector mesons, then $k = 7/291 \approx 1/40$. To get the true answer it is necessary to take into account the interaction between the particles at this temperature and at the corresponding density, something still impossible at the present time.

$$\frac{\tau}{t} = \frac{6m}{k\,(2m+3)}.$$

Under thermodynamic equilibrium, the values of $\varepsilon_\nu/\varepsilon$ k at different temperatures range from $k = 1/4$ to $k = 1/40$. Consequently, Misner must use the viscosity concept at $\tau/t \approx 5$ or even $\tau/t \approx 50$. It is obvious that the formal use of the viscosity concept is not valid at so long a free path: the viscosity describes the first term of the expansion of the exact equations in terms of τ/t. When τ/t is not small, it is necessary to solve the kinetic equation for weakly interacting particles. However, rejection of the macroscopic description of the phenomenon with the aid of viscosity still does not mean that Misner's conclusions are qualitatively incorrect.

We can seek a solution of the kinetic equation having the same properties as Misner's solution. Such a solution is self-similar, meaning that the neutrino distribution function in momentum space depends only on the dimensionless ratios cp_x/T, cp_y/T, and cp_z/T, and the temperature has a power-law variation $T \sim t^{-1/5}$. This form of the solutions is compatible with the structure of the equations and makes it possible to simplify greatly the solution of the equations and reduce it to quadratures.

Before we analyze this possibility, we note the following: If the parameters of the anisotropic model are such that the free path time of the neutrino becomes larger than t at a high temperature, when many particles are in equilibrium and $k = \varepsilon_\nu/\varepsilon = 1/20 - 1/40$, then it can be seen even without a detailed analysis of the kinetic equation that the regime described in [1,2] sets in.

Indeed, it follows from (1) that at small values of k the Misner regime does not set in even by the instant when $\tau/t \approx 1$ and the viscosity does not change the energy density $\varepsilon = \varepsilon_a$ noticeably. After this instant, the approximation of "free" particles is already applicable; the ratio of the neutrino momentum along one of the axes, along which the compression takes place, to the momenta along the other axes, increases as a result of the blue shift, and the entropy increases as a result of the processes that were described in [1,2] for the case $m = 1$. Thus, at small values of k and for real $m = 1$, the Misner regime does not set in. The answer for all values of k (both large and small) is given by the solution of the kinetic equation.

The kinetic equation was analyzed for two model problems: one in which only processes of single creation and annihilation of neutrinos were taken into account, and the other with paired production and annihilation, the case of scattering has not yet been considered. It was assumed that $m = 1$ and that the exponents in the cosmological solution are +2/3, +2/3, and -1/3. The analysis will be described in detail in another paper. We present here its results. It turned out that the self-similar solution (indicated above), which generalizes Misner's viscous regime $T \sim t^{-1/5}$, exists only at a very large and physically unreal value of the dimensionless parameter $k = \varepsilon_{\nu eq}/\varepsilon_{eq} > 0.98$, which characterizes the thermodynamic properties of matter. Here $\varepsilon_{\nu eq}$ and ε'_{eq} are respectively the equilibrium density of all types of neutrino and the equilibrium density of all types of particles (including the neutrinos) in the energy region of interest to us.

Actually, as already noted, we should expect $\nu_{eq}/\nu_{eq} < 0.25$. Under these conditions, the self-similar solution does not exist, meaning not only that the viscosity concept cannot be used, but also that there is no solution similar to the viscous one.

The physical conclusion is that when deviation from equilibrium of the weakly-interacting particles begins in the anisotropic cosmological solution, then these particles soon become free, their number decreases, and the average energy increases in the manner described in [1, 2].

It is seen from the foregoing that if the early stages of the expansion were actually described by the Heckmann-Schucking model [5], then there should now exist, according to [1, 2], a directed flux of neutrinos of energy $E \approx 10^4$ eV and density $\varepsilon_\nu \approx 10^{-12}$ erg/cm^3, which in principle should be experimentally verifiable.

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INFLUENCE OF MULTIPLE SCATTERING ON RESONANT RADIATION

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When charged particles move uniformly in a medium having a periodically varying electron density, the radiation produced by different inhomogeneities can add up coherently only if certain phase relations are satisfied; these are very similar to the corresponding condition for the appearance of Cerenkov radiation in a homogeneous medium [1]. Actually a nonrelativistic particle moving uniformly along the variation of the properties of the medium will emit frequencies that are integer multiples of the frequency at which the period of the medium is traversed. In the case of relativistic particles it is necessary to take into account also the Doppler shift, and the necessary condition for the appearance of radiation (the resonance condition) can be written in the form

$$\omega\left(1 - \frac{v}{c}\sqrt{\epsilon}\cos\theta\right) = \frac{2\pi v}{\ell}r. \tag{1}$$

Here v is the particle velocity, θ the quantum emission angle, ℓ the period of the medium, r an integer, and ε the dielectric constant of the medium.

If the particle is scattered, then condition (1) (which corresponds to the law of conservation of the longitudinal component of the momentum) is violated, inasmuch as