

neutrinos) in the energy region of interest to us.

Actually, as already noted, we should expect  $v_{eq}/v_{eq} < 0.25$ . Under these conditions, the self-similar solution does not exist, meaning not only that the viscosity concept cannot be used, but also that there is no solution similar to the viscous one.

The physical conclusion is that when deviation from equilibrium of the weakly-interacting particles begins in the anisotropic cosmological solution, then these particles soon become free, their number decreases, and the average energy increases in the manner described in [1, 2].

It is seen from the foregoing that if the early stages of the expansion were actually described by the Heckmann-Schucking model [5], then there should now exist, according to [1, 2], a directed flux of neutrinos of energy  $E \approx 10^4$  eV and density  $\epsilon_\nu \approx 10^{-12}$  erg/cm<sup>3</sup>, which in principle should be experimentally verifiable.

The authors thank A. N. Shvarts for help.

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#### INFLUENCE OF MULTIPLE SCATTERING ON RESONANT RADIATION

M. L. Ter-Mikaelyan

Physics Research Institute, Armenian Academy of Sciences

Submitted 21 May 1968

ZhETF Pis. Red. 8, No. 2, 100-102 (20 July 1968)

When charged particles move uniformly in a medium having a periodically varying electron density, the radiation produced by different inhomogeneities can add up coherently only if certain phase relations are satisfied; these are very similar to the corresponding condition for the appearance of Cerenkov radiation in a homogeneous medium [1]. Actually a nonrelativistic particle moving uniformly along the variation of the properties of the medium will emit frequencies that are integer multiples of the frequency at which the period of the medium is traversed. In the case of relativistic particles it is necessary to take into account also the Doppler shift, and the necessary condition for the appearance of radiation (the resonance condition) can be written in the form

$$\omega \left(1 - \frac{v}{c} \sqrt{\epsilon} \cos \theta\right) = \frac{2\pi v}{\ell} r. \quad (1)$$

Here  $v$  is the particle velocity,  $\theta$  the quantum emission angle,  $\ell$  the period of the medium,  $r$  an integer, and  $\epsilon$  the dielectric constant of the medium.

If the particle is scattered, then condition (1) (which corresponds to the law of conservation of the longitudinal component of the momentum) is violated, inasmuch as

in multiple scattering the momentum can also be transferred to individual nuclei of the medium. The reciprocal of the minimum longitudinal momentum determines the coherent length, i.e., the length of the particle trajectory from which the photons are emitted. Using (1), we get in the limit of high frequencies and relativistic energies

$$l_{\text{coh}} = \frac{2c}{\omega \left( 1 - \frac{v^2}{c^2} + \frac{\omega_0^2}{\omega^2} - \frac{2\pi r v}{l\omega} \right)} \quad (2)$$

where  $\omega_0$  is the Langmuir frequency. The coherent length introduced in this manner should be understood somewhat arbitrarily, since it assumes a negative value for the region of the resonantly emitted frequencies. We must now formulate a criterion for the influence of the multiple scattering on the resonant radiation [2]. If the mean square of the angle of multiple scattering over the coherent length (we naturally choose the absolute value of  $l_{\text{coh}}$ ) is of the same order as or larger than the characteristic angle of emission of the resonant quanta, the order of which is  $\theta_{\text{rad}}^2 = 4\pi v r / l$ , then we can expect an appreciable modification of the earlier results. These conditions were satisfied in the experiments performed by Arutyunyan and co-workers [3, 4], which stimulated the performance of the present work. The influence of the scattering is manifest in two ways: on the one hand, the scattering disturbs the coherence, and the resonant radiation proper is consequently decreased; on the other hand, the scattering leads to the appearance of bremsstrahlung. We calculate the radiation using a periodic sinusoidal medium as an example. We assume that the density of the nuclei is constant, and the change of the dielectric constant is effected by oscillations of the electron density. Using Migdal's procedure [5] to take into account the influence of the multiple scattering, we obtain for the energy of frequency  $\omega$  radiated per unit time, on moving in a medium having a dielectric constant

$$\epsilon = \epsilon_0 + \Delta \cos \frac{2\pi Z}{l} \quad (3)$$

the expression

$$dI_{\omega} = - \frac{2e^2 \omega}{\pi} \sum_{r=-\infty}^{r=+\infty} \frac{|I_r(B)|^2}{l_{\text{coh}}} \Phi(S), \quad (4)$$

where  $I_r(B)$  is a Bessel function of order  $r$ , with an argument  $B = l\Delta\omega/4\pi c$ . The function  $\Phi(S)$  for positive  $S$  is connected with the function introduced by Migdal in [5] by the relation  $\Phi(S) = -\phi_M(S)/24S^2$ .  $S$  is equal to

$$S = \frac{c}{4l_{\text{coh}}\sqrt{\omega q}}, \quad (5)$$

where  $q = E_S^2 c / |8E^2|$  is the mean square of the angle of multiple scattering per cm, multiplied by  $c/\beta$ . The absolute value of  $S$  is of the order of the ratio of the emission angle to the angle of multiple scattering over the coherent length. Consequently, if  $S$  is much larger than unity, the deviations from the previously obtained formulas [1,2] will be small. Indeed,

using the expansion of  $\phi(S)$  for  $|S| \gg 1$ , we get

$$dI_{\omega}(|S| \gg 1) = - \frac{2e^2 d\omega}{\pi} \left\{ \sum_{r=-\infty}^{r=+\infty} \frac{|I_r(B)|^2}{\ell_{\text{coh}}} \right\} - \frac{\pi}{2} \text{sign} \ell_{\text{coh}} + \frac{\pi}{2} - \left\{ - \frac{2}{3} \frac{q\omega \ell_{\text{coh}}^2}{c^2} \right\} \quad (6)$$

The region of negative  $\ell_{\text{coh}}$  is the region of resonant quanta, and the first two terms in the curly brackets lead to the formula obtained earlier with scattering neglected; the third term is always negative and decreases the intensity of the resonant quanta.

The region of positive  $\ell_{\text{coh}}$  is the region of bremsstrahlung quanta, and the third term in the curly brackets leads in this case to a positive contribution to the radiation. The other limiting case of formula (4) corresponds to large scattering. The limiting formula is

$$dI_{\omega}(|S| \ll 1) = \frac{2e^2}{\pi c} \sqrt{q\omega} d\omega \sum_{r=-\infty}^{r=+\infty} |I_r(B)|^2. \quad (7)$$

If the condition  $|S| \ll 1$  is satisfied for all harmonics, then summation of the Bessel functions yields an exact formula for the intensity of the bremsstrahlung in the limit of high energies, with allowance for both multiple scattering and the polarization of the medium.

Depending on the concrete experiment, the total emission of photons of given frequency in a periodic medium can be either increased or decreased by the influence of the multiple scattering. Formulas of similar type can be readily obtained also for other periodic media.

The author is grateful to E. L. Feinberg, I. M. Frank, V. L. Ginzburg, F. R. Arutyunyan, and M. I. Ryazanov for kind discussions.

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#### MAGNETIC RESONANCE IN ANTIFERROMAGNETIC METALS IN A MAGNETIC FIELD

A. Ya. Blank and P. S. Kondratenko  
 Institute of Radiophysics and Electronics, Ukrainian Academy of Sciences  
 Submitted 27 May 1968  
 ZhETF Pis. Red. 8, No. 2, 103 - 105 (20 July 1968)

The spectrum of the magnetic excitations of an antiferromagnetic metal is formed by the collective interactions in the system of conduction electrons and magnetic sublattices. Consequently, neither the antiferromagnetic-resonance frequencies due to the interaction between the sublattices (as in a dielectric), nor the natural frequencies of the electron subsystem (as in paramagnetic metals) represent the spectrum of the magnetic excitations of the antiferromagnetic metal as a whole. In this communication we present certain results pertaining to the magnetic spectrum of an antiferromagnetic metal.

We regard the antiferromagnetic metal as a system of two mirroring ionic magnetic