using the expansion of $\Phi(S)$ for $|S| \gg 1$, we get

$$dI_\omega(|S| \gg 1) = -\frac{2e^2 d\omega}{\pi} \sum_{r=-\infty}^{r=+\infty} \frac{|I_r(B)|^2}{l_{coh}} \left\{ -\frac{\pi}{2} \operatorname{sign} l_{coh} + \frac{\pi}{2} - \frac{2}{3} \frac{q\omega l_{coh}^2}{c^2} \right\} \tag{6}$$

The region of negative l_{coh} is the region of resonant quanta, and the first two terms in the curly brackets lead to the formula obtained earlier with scattering neglected; the third term is always negative and decreases the intensity of the resonant quanta.

The region of positive l_{coh} is the region of bremsstrahlung quanta, and the third term in the curly brackets leads in this case to a positive contribution to the radiation. The other limiting case of formula (4) corresponds to large scattering. The limiting formula is

$$dI_\omega(|S| \ll 1) = \frac{2e^2}{\pi c} \sqrt{q\omega} d\omega \sum_{r=-\infty}^{r=+\infty} |I_r(B)|^2. \tag{7}$$

If the condition $|S| \ll 1$ is satisfied for all harmonics, then summation of the Bessel functions yields an exact formula for the intnesity of the bremsstrahlung in the limit of high energies, with allowance for both multiple scattering and the polarization of the medium.

Depending on the concrete experiment, the total emission of photons of given frequency in a periodic medium can be either increased or decreased by the influence of the multiple scattering. Formulas of similar type can be readily obtained also for other periodic media.

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MAGNETIC RESONANCE IN ANTIFERROMAGNETIC METALS IN A MAGNETIC FIELD

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The spectrum of the magnetic excitations of an antiferromagnetic metal is formed by the collective interactions in the system of conduction electrons and magnetic sublattices. Consequently, neither the antiferromagnetic-resonance frequencies due to the interaction between the sublattices (as in a dielectric), nor the natural frequencies of the electron subsystem (as in paramagnetic metals) represent the spectrum of the magnetic excitations of the antiferromagnetic metal as a whole. In this communication we present certain results pertaining to the magnetic spectrum of an antiferromagnetic metal.

We regard the antiferromagnetic metal as a system of two mirroring ionic magnetic

sublattices with anisotropy of the "easy axis" type and a Fermi liquid of s-electrons. The magnetic field is applied along the antiferromagnetism axis.

The spectrum of the magnetic excitation of this system is determined by the singularities of the component, transverse to the spin, of the two-particle vertex part $\Gamma_{\uparrow\downarrow\uparrow\downarrow}$ or $\Gamma_{\downarrow\uparrow\uparrow\downarrow}$ of the electrons. We analyzed the magnetic spectrum of the metal in the long-wave limit. At $\vec{k} = 0$ ($\vec{k}$ = magnon momentum), the natural frequencies of the entire system are determined by the equations

$$\omega_{lm}(H) = \left(\frac{2\mu_0 H}{1+B_0} + m\Omega\right)(1+B_l) \quad l \geq 1, \ |m| \leq l, \tag{1}$$

$$(\omega - 2\mu_0 H)[(\omega - 2\mu_0 H)^2 - \tilde{\omega}_s^2] = \xi\tilde{\omega}_s^2 2\mu_0 H. \tag{2}$$

Here H is the external magnetic field, Ω the cyclotron frequency, μ_0 the Bohr magneton, $B_l = \zeta_l \nu/(2l+1)$, ν is the state density of the s-electrons on the Fermi surface, ζ_l the coefficient of expansion of the exchange part of the Landau function [1], $\tilde{\omega}_s$ the renormalized frequency of the antiferromagnetic resonance in the absence of a magnetic field [2], $\xi = (\tilde{\omega}_s^2 - \omega_0^2)/\tilde{\omega}_s^2$ a parameter characterizing the exchange interaction of the s-electrons with the magnetic sublattices, and ω_0 the frequency connected with the gapless branch of the spin waves at $\vec{H} = 0$ [2].

When $\vec{k} = 0$, the natural frequencies of the antiferromagnetic metal are the two-parameter family of resonant frequencies ω_{lm} ($l \neq 0$), which are essentially homogeneous spin waves in a paramagnetic metal [3], and the three resonant frequencies given by Eq. (2). As a result of the exchange interaction of the electrons with the sublattices, the isotropic harmonic with frequency ω_{00} equal to the parmagnetic-resonance frequency is connected with the antiferromagnetic frequencies ω_s. The last three frequencies depend in nonlinear fashion on the magnetic field. At small $2\mu_0 H/\tilde{\omega}_s$ we have

$$\omega_1 = (1-\xi)2\mu_0 H - \xi^3 2\mu_0 H(2\mu_0 H/\tilde{\omega}_s)^2, \tag{3}$$

$$\omega_{2,3} = \pm\tilde{\omega}_s + (1+\frac{\xi}{2})2\mu_0 H \mp \frac{3}{8}\xi^3 2\mu_0 H\left(\frac{2\mu_0 H}{\tilde{\omega}_s}\right)^{1)}. \tag{4}$$

It can be seen from (2) that when H increases the frequency ω can become equal to zero. The vanishing of ω at a certain field $H = H_k$ corresponds to an infinite static magnetic susceptibility of the system (transverse to the field), thus pointing to a phase transition into a state with noncollinear magnetic moments. According to (2) we get

$$H_k = \frac{\tilde{\omega}_s}{2\mu_0}\sqrt{1-\xi}. \tag{5}$$

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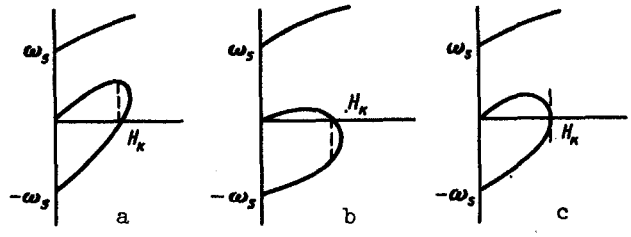
1) The positive and negative solutions of the dispersion equation correspond, as is well known, to different wave polarizations. The excitation energy ε_s is connected in this case with the frequency by the relation $\varepsilon_s = |\omega_s|$. Bearing this in mind, we shall use also the negative frequencies.

Depending on the value of the parameter ξ, both the "antiferromagnetic" frequency (ω_3) and the "paramagnetic" frequency (ω_1) may vanish. The former case corresponds to $\xi < 2/3$ and the latter to $\xi > 2/3$. The figure shows schematically the dependence of ω on H. Of particular interest is the case when ξ is close to unity ($1 - \xi \ll 1$). In this case the phase transition will take place in fields that are much smaller than the critical field of the dielectric, i.e., even relatively weak magnetic fields will disturb the antiferromagnetic order in the lattice. Thus, the exchange interaction of the s-electrons determines to a considerable degree the magnetic structure of the system.



We note that when $\xi \sim 1$ the formulas in (3) are valid in the entire region of existence of antiferromagnetic order.

As to the possible experimental observation of the considered effects, we note the following. The spectrum of magnetic excitations in nonferromagnetic metals has by now become a realistic subject of experimental investigations. We have in mind magnetic excitations in an electron system - the recent observation of spin waves in paramagnetic metals [4], and also observation of weakly damped electromagnetic waves due to Fermi-liquid interaction near cyclotron resonance [5]. Owing to the existence of magnetic order, an antiferromagnetic metal is, generally speaking, a very convenient object for the study of the role of a Fermi liquid of electrons in the magnetic properties of the metal. However, the conventional model of an antiferromagnetic metal, which we employ at present, is not adequate for a description of d-metals (such as chromium) in which the d-electrons are not strictly localized. Everything said above can apply more readily to rare-earth metals, in which it is possible to separate an ionic subsystem having antiferromagnetic order.

The derivation of the present results, including an analysis of the spectrum of the magnetic excitations when $\vec{k}$ differs from zero, is the subject of a separate publication.

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PHENOMENOLOGICAL THEORY OF K^0 MESONS AND THE NON-EXPONENTIAL CHARACTER OF THE DECAY

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1. In the well-known phenomenological K^0-meson theory (see, e.g., [1-3]), unitarity relations and their various consequences were obtained on the basis of the most general premises of quantum theory, assuming that the decay of the K_L or K_S meson is strictly exponential.