Pis'ma v ZhETF, vol. 86, iss. 3, pp. 190 – 192

About the role of vector mesons on the $\eta \to \pi^0\gamma\gamma$ decay width in meson – baryon chiral model

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Submitted 13 June 2007

It is shown in the work of one of the authors in 1979 (MKV) that the contribution to the amplitude of this decay from diagrams with one baryon loop is equal to zero and contributions from diagrams with meson loops appear very small. However, pole diagrams with intermediate vector mesons were not considered there. Here it is shown that contributions of these pole diagrams dominate. The meson-baryon chiral model used here is compared with known quark chiral models. The obtained results are in satisfactory agreement with recent experimental data.

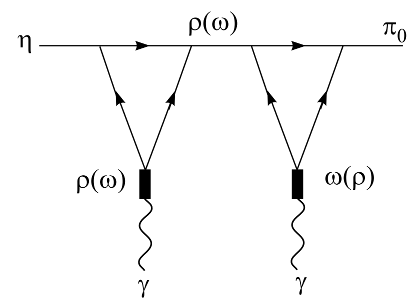
PACS: 11.30.Rd, 13.20.–v, 13.25.–k

The investigations of the process $\eta \to \pi^0\gamma\gamma$ have a long history [1 – 8] (see in detail [1]). The first theoretical estimates of a depth of this process arose in the sixties of the last century [1]. The first experimental data were based on a few statistics and they were not precise. So these results gave arguments to predict a large value of the branching ratio of the process 26 eV. Critical arguments of this prediction can be found in [2]. A real breakthrough in the investigation of this process happened in the GAMS experiment in 1981 at Protvino [3] where the value $\Gamma_{\eta\to\pi\gamma\gamma} = 0.84 \pm 0.18\,\mathrm{eV}$ was obtained. Notice that this result is consistent with those obtained in the Nambu – Jona-Lasinio model [4, 5].

At the last time, attention to this process revived from both a theoretical and experimental points of view (see [7] and cites in it). It was promoted first of all by careful measurement in the CERN laboratory of the $\eta \to 3\pi$ and $\eta \to \pi_0\gamma\gamma$ decay probabilities, where about one million events was already investigated. As a result, the estimation of the width decreased in comparison with Prokoshkin's estimation [3] almost twice. During the same time theoretical works were published in which an attempt to explain these results was undertaken. In these works [6] vector dominance models, various chiral quark models or a model with meson loops were used in the framework of Chiral Perturbation Theory (ChPT) with decomposition into momenta down to the eighth order p^8.

Let us pay attention to that in quark chiral models there is a number of basic lacks. Namely, by consideration of quark loops the quark confinement principle was not provided. Ultra-violet divergence was eliminated by introduction of a limiting momentum. In using linear sigma models there is a problem of explanation of the big mass $a_0(980)$ meson in the framework of both the chiral quark model and Nambu – Jona-Lasinio (NJL) one [5].

By virtue of the above – said of certain interest is the description of this process by using the chiral meson-baryon Lagrangian by analogy with the works of 1978–1979 [2, 9]. In [2], the contribution of intermediate vector ω and ρ-mesons (see figure) however was not considered.



Feynman diagrams with intermediate vector mesons

The absence of confinement problems is thought to be the advantage of the chiral meson-baryon model. An important point is the absence of ultra-violet divergences, which is a consequence of gauge invariance of the model. Further, in view of large baryon masses in calculating amplitudes corresponding to loop diagrams, it is possible to use only the lowest terms of decomposition into external momenta. We show in this work the approximate validity of the principle of quark-hadron duality (see also paper of Volkov and Osipov in ref. [6]). In the description in terms of quark models indepen-

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dence of triangular amplitudes of the constituent quark mass is used and the factor three-number colors is introduced. In our approach it is necessary to use the Goldberger-Treimann relation for a constant of meson-baryon coupling $g = g_A M_B/F_\pi, F_\pi = 93\,\mathrm{MeV}$, which includes the constant of renormalization of a strong vertex top in the neutron β decay $g_A = 1,25$, consider contributions of charged baryons from an octet with the corresponding factors following from $SU(3)$ symmetry and consider their various masses. Dependence on the baryon masses M_B in amplitudes corresponding to diagrams of abnormal type disappears as in the case of quarks. As a result, we obtain the factor 3.3 instead of the color factor 3 quarks accepted in the model for the triangular diagram $\pi_0\rho\omega$.

This "almost full" coincidence can be considered as the proof of the validity of the principle of the quark-hadron duality. We show that within the framework of the realistic chiral-symmetric baryon-meson model, where physical observable objects are only used, it is possible to get a satisfactory description of radiation decays of vector mesons and a rare process $\eta \to \pi_0\gamma\gamma$ in accord with recent experimental data. It is the motivation of our work.

In the beginning, using part of the baryon-meson interaction Lagrangian, we calculate correcting factors for the anomalous triangular diagrams which will play an important role in the following calculation. Using these factors and results obtained within the quark models, we describe the widths of radiation $\rho(\omega) \to \pi_0(\eta)\gamma$ vector meson decays. Also, we compare these widths with modern experimental data. Further we will consider double radiation η decay $\eta \to \pi_0\gamma\gamma$. In the conclusion, discussion of the obtained results is given.

Part of the Lagrangian describing interaction of an octet of baryons with neutral pion and η-meson looks like [2, 9]

$$L_{P\bar{N}N} = \\ =ig\pi_0\left[\bar{p}\gamma_5 p+\frac{1}{3}\bar{\Xi}^-\gamma_5\Xi^-+\frac{2}{3}(\bar{\Sigma}^+\gamma_5\Sigma^+-\bar{\Sigma}^-\gamma_5\Sigma^-)\right]+ \\ i\frac{g}{\sqrt{3}}\eta_8\left[\frac{1}{3}\bar{p}\gamma_5 p-\frac{5}{3}\bar{\Xi}^-\gamma_5\Xi^-+\frac{4}{3}(\bar{\Sigma}^+\gamma_5\Sigma^++\bar{\Sigma}^-\gamma_5\Sigma^-)\right]+ \\ +ig\eta_0\left[\bar{p}\gamma_5 p+\bar{\Xi}^-\gamma_5\Xi^-+\bar{\Sigma}^+\gamma_5\Sigma^++\bar{\Sigma}^-\gamma_5\Sigma^-\right], \quad (1)$$

where π_0 is the pion field, η_0, η_8 are the singlet and octet components of η-meson, $p, \Xi^-, \Sigma^\pm$ are the baryon octet. For completeness let us also give Lagrangians of interactions of vector mesons with baryons

$$L_{V\bar{N}N} = \\ =i\frac{g_\rho}{2}[\rho_\mu\left(\bar{p}\gamma_\mu p-\bar{\Xi}^-\gamma_\mu\Xi^-+\bar{\Sigma}^+\gamma_\mu\Sigma^+-\bar{\Sigma}^-\gamma_\mu\Sigma^-\right)+ \\ +\omega_\mu\left(\bar{p}\gamma_\mu p+\bar{\Xi}^-\gamma_\mu\Xi^-+\bar{\Sigma}^+\gamma_\mu\Sigma^++\bar{\Sigma}^-\gamma_\mu\Sigma^-\right)], \quad (2)$$

and ones describing photon-vector meson transitions (vector dominance model)

$$L_{VA} = \frac{e}{g_\rho}A_\mu\left[m_\rho^2\rho_\mu+\frac{1}{3}m_\omega^2\omega_\mu+\frac{\sqrt{2}}{3}m_\phi^2\phi_\mu\right], \quad (3)$$

where A_μ is a photon field, and $\rho_\mu, \omega_\mu, \phi_\mu$ are the nonet vector fields.

Let us calculate correcting factors which are to be considered in transition from quark models to the model with baryons. For the triangular diagram with external $\pi\rho\omega$, using the resulted formulas, we have

$$K_\pi = \frac{1}{3}g_A\left[1+\frac{1}{3}+\frac{4}{3}\right] = 1.1. \quad (4)$$

For the vertex functions $\eta\omega\omega, \eta\rho\rho$ we get

$$K_{\eta_8} = \frac{1}{3\sqrt{3}}g_A\left[1-\frac{5}{3}+\frac{8}{3}\right] = 0.326; \\ K_{\eta_0} = \frac{1}{3}g_A[1+1+2] = 1.68. \quad (5)$$

As a result, for real η-meson, taking into account the singlet-octet mixing

$$\eta = \eta_0\cos(\theta_0-\theta)+\eta_8\sin(\theta_0-\theta),$$

where an ideal mixing angle is $\cos\theta_0 = \sqrt{2/3}$, and choosing a deviation from an ideal angle $\theta = -18^0$, we obtain

$$K_\eta = 1.27. \quad (6)$$

Expressions for the decay widths of vector mesons, given in [5], in view of the correcting factors look like

$$\Gamma^{\rho\to\pi_0\gamma} = K_\pi^2\frac{2\pi\alpha}{3}\frac{g_\rho^2}{4\pi}\frac{1}{(16\pi^2F_\pi)^2}\left(\frac{M_\rho^2-M_\pi^2}{M_\rho}\right)^3, \\ \Gamma^{\omega\to\pi_0\gamma} = 9\Gamma^{\rho\to\pi_0\gamma}; \quad (7)$$

$$\Gamma^{\rho\to\eta\gamma} = K_\eta^2\pi\alpha\frac{g_\rho^2}{4\pi}\frac{1}{(16\pi^2F_\pi)^2}\left(\frac{M_\rho^2-M_\eta^2}{M_\rho}\right)^3, \\ \Gamma^{\omega\to\eta\gamma} = \frac{1}{9}\Gamma^{\rho\to\eta\gamma}. \quad (8)$$

Substituting the above factors and also $F_\pi = 93\,\mathrm{MeV}$, $g_\rho = 5.94$ we have

$$\Gamma_{th}^{\rho\to\pi_0\gamma} = 93.5\,\mathrm{KeV}; \quad \Gamma_{th}^{\omega\to\pi_0\gamma} = 930\,\mathrm{KeV}; \\ \Gamma_{th}^{\rho\to\eta\gamma} = 42.8\,\mathrm{KeV}; \quad \Gamma_{th}^{\omega\to\eta\gamma} = 4.96\,\mathrm{KeV}. \quad (9)$$

For comparison we write out the results of experiments (PDG-2006):

$$\begin{aligned} \Gamma_{\rm exp}^{\rho\to\pi_0\gamma} &= (89.4 \pm 12)\,\text{KeV}; \\ \Gamma_{\rm exp}^{\omega\to\pi_0\gamma} &= (756,5 \pm 21)\,\text{KeV}; \\ \Gamma_{\rm exp}^{\rho\to\eta\gamma} &= (44.7 \pm 4.5)\,\text{KeV}; \\ \Gamma_{\rm exp}^{\omega\to\eta\gamma} &= (4.17 \pm 0.5)\text{KeV}. \end{aligned} \tag{10}$$

Comparing these results, we can see that they are in satisfactory agreement with each other. We now consider the process $\eta \to \pi_0\gamma\gamma$.

As shown in work [2] the contributions of amplitudes to Feynman diagrams corresponding to one-baryon loop with three and four vertices within the limits of chiral baryon models precisely compensate each other, the contribution of diagrams with mesons in the closed loops is the value of an order of $\Gamma_{(\pi)} \approx 0.01$ eV and, thus, is very small.

Hence it follows that in describing the decay $\eta \to \to \pi_0\gamma\gamma$ the dominating contribution comes from the diagrams with intermediate ρ, ω mesons.

Calculation of the corresponding contribution is close to a similar calculation within the framework of quark models [5]. In [5] the factor $\cos\theta - \sqrt{2}\sin\theta \approx 1.39$ was used for $\theta = -18^0$ which describes the singlet-octet mixing for η-meson. It is necessary to replace this factor by $K_\eta = 1.27$, obtained above. Using the results of numerical calculations [5] we get for the width of double radiation decay of this meson

$$\Gamma_{th}^{\eta\to\pi_0\gamma\gamma} = (0.326 \pm 0.02)\ \text{eV}. \tag{11}$$

The error estimation of our result comes from the uncertainties of used parameters. This result is in satisfactory agreement with the result of experimental work [8]

$$\Gamma_{exp}^{\eta\to\pi_0\gamma\gamma} = (0.45 \pm 0.12)\ \text{eV}. \tag{12}$$

We are grateful to S.B. Gerasimov and A.E. Dorokhov for useful discussions, V. Bytev and N. Kornakov for help. One of us (MKV) acknowledges the support of the Russian Foundation for Basic Research # 05-02-16699, EAK is grateful to the INTAS-05-1000008-8328.

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