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CONDUCTIVITY OF SEMICONDUCTORS UNDER PINCH-EFFECT CONDITIONS

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We shall derive in this paper the current-voltage characteristics of semiconductors in the presence of the pinch effect, as functions of different carrier-scattering mechanisms.

The equation describing the contraction of an electron-hole plasma, assuming $n_e \approx n_h = n \gg N_i$, has in cylindrical coordinates the form [1]

$$\frac{dn}{dr} + \frac{4\pi e}{c^2} E_z^2 \frac{\mu_e \mu_h (\mu_e + \mu_h)}{D_e \mu_h + D_h \mu_e} \frac{n}{r} \int_0^r n(\mu_e + \mu_h) r' dr' - \frac{\mu_e + \mu_h}{D_e \mu_h + D_h \mu_e} \frac{1}{r} \int_0^r (P - g) r' dr' = 0 . \tag{1}$$

Here n is the electron or hole density, N_i is the density of the charged impurities, e is the electron charge, c is the velocity of light, $\mu_{e,h}$ and $D_{e,h}$ are the mobilities and diffusion coefficients of the electrons and holes, E_z is the constant electric field, and P and g are the recombination and generation rates, respectively.

A strongly non-equilibrium plasma with $n \gg N_i$ can be obtained with the aid of two photon absorption or injection. We shall assume the recombination to be either linear in the concentration, $P = n/\tau$, where τ is the carrier lifetime, or else quadratic $P = \gamma n^2$.

To find the dependence of the current on the field it is necessary to find the function n(r) for the different scattering mechanisms that determine the electron and the hole mobilities. It is impossible to solve (1) in a general case. At low generation and recombination rates, the third term in (1) can be discarded when determining the n(r) dependence, but it must be taken into account in the determination of the average density $\bar{n}$. The conditions for the smallness of the third term of equation (1) in the case of linear and quadratic recombination are, respectively,

$$\tau \gg \frac{c^2}{4\pi e \mu_e \mu_h (\mu_e + \mu_h) E_z^2 \bar{n}} , \quad \gamma \ll \frac{4\pi e \mu_e \mu_h (\mu_e + \mu_h) E_z^2}{c^2} . \tag{2}$$

For InSb at T ~ 100°K, $\mu_e \sim 10^5$ cm^2/V-sec, $\mu_h \sim 10^4$ cm^2/V-sec, and $E_z \sim 100$ V/cm condition (2) assumes the form $\tau \gg 10^7/\bar{n}$ sec and $\gamma \ll 10^{-8}$ cm^{3}sec^{-1}. If it is recognized that in InSb we have [2] $\tau \sim 10^{-6} - 10^{-7}$ sec and $\gamma \lesssim 10^{-9} cm^3 sec^{-1}$, condition (2) seems quite realistic.

1. Nondegenerate electron-hole gas. The scattering of the carriers by phonons [3, 4], and by one another [5] was investigated earlier. We consider the case of combined scattering, when the electrons are scattered by holes and the holes are scattered by phonons. It has been shown both experimentally [6] and theoretically [7] that this mechanism predominates in InSb at concentrations $n > 10^{14}$ cm^{-3} ($T \sim 100°K$). In this case the mobility of the electrons is inversely proportional to the concentration [8], and the mobility of the holes is constant.

Taking the foregoing into account, we obtain from (1)

$$\left.\begin{aligned} n(r) &= \frac{\bar{\mu}_e}{\mu_h}\bar{n}\left[\frac{(1+\eta)^2}{\left(1+\eta\frac{r^2}{a^2}\right)^2}-1\right], \quad a = R\sqrt{\frac{\mu_h}{\mu_e}\frac{1}{\eta}}, \quad \eta = \frac{\xi}{1-\xi} \\ \xi &= \sqrt{\pi R^2\beta\bar{n}}, \quad \beta = \frac{e^2\mu_h^2E_z^2}{4c^2KT}, \quad I = 2cR\sqrt{\pi\bar{n}kT} \end{aligned}\right\}. \tag{3}$$

Here a is the radius of the pinch and I is the current through a sample of radius R. All quantities pertaining to linear and quadratic recombinations will henceforth be designated by the indices (1) and (2)

$$\bar{n}^{(1)} = gr, \quad \bar{n}^{(2)} = \frac{1}{\pi R^2\beta}\left[1-\sqrt{\frac{\gamma\bar{n}\bar{\mu}_e}{3\pi R^2\beta g\mu_h}}\right]^2. \tag{4}$$

In formulas (3) and (4), $\bar{\mu}_e$ is the mobility at the average concentration (the superior bar denotes quantities determined at the average concentration), and consequently $\bar{n}\bar{\mu}_e$ does not depend on the average concentration. Expressions (3) and (4) are valid in the case when the radius of the pinch is smaller than the radius of the sample, i.e., in fields $E_z > E_{zc}$, where $\eta(E_{zc}) = \mu_h/\bar{\mu}_e$. When $R \sim 10^{-1}$ cm and $\bar{n} \sim 10^{15}$ cm^{-3} we have $E_{zc}^{(1,2)} \sim 10$ V/cm.

It follows from (3) and (4) that in the case of linear recombination the pinch effect leads to saturation of the current (when the carriers are scattered by phonons, we obtain Ohm's law), and in the case of quadratic recombination ($I \sim E_z^{-1}$) we obtain a decreasing characteristic when $E_z > E_{zc}^{(2)}$ (in the case of scattering by phonons we get $I \sim E_z^{-1}$).

2. Degenerate electron-hole gas. The degeneracy changes the picture of the pinch effect radically: the Einstein relation must be replaced by the relation $D = (2/3)(\varepsilon_F/e)\,\mu$, where ε_F is the Fermi energy, and the mobility becomes dependent on the Fermi energy and consequently on the concentration[9], $\mu = \bar{\mu}\,(n/\bar{n})^{2q/3}$, where q assumes different values for different scattering mechanisms. In this case it is impossible to solve (1) analytically at certain values. However, all the qualitative relations can be determined. Let us introduce the dimensionless variables

$$Y = (n/n_m)^{2/3(1-q)}, \quad t = (1-q)\frac{\pi e^2E_z^2}{c^2\bar{n}^{2/3}(2q-1)}\frac{(\bar{\mu}_e+\bar{\mu}_h)^2}{\bar{\varepsilon}_{Fe}+\bar{\varepsilon}_{Fh}}n_m^{4/3\,q+1/3}r^2,$$

$$E_{zc}=\frac{c\sqrt{\epsilon_{Feo}+\epsilon_{Fho}}}{e(\mu_{eo}+\mu_{ho})\sqrt{|1-q|\pi R^2 n_o}}\,, \tag{5}$$

where $n_m = n\ (r = 0)$; n_0, μ_0, and ϵ_{F0} are the concentration, mobility, and Fermi energy in the absence of a field, $n_0^{(1)} = g\tau$, and $n_0^{(2)} = \sqrt{g/\gamma}$.

In the notation of (5) Eq. (1) reduces to an equation without parameters and with boundary conditions $Y(t = 0)$ and 1 and $Y'(t = 0) = -1$. Its solution is a certain numerical function (independent of any parameter), which vanishes when $t = t_q \sim 1$. From this, using (5), we obtain, accurate to numerical coefficients of the order of unity

$$\bar{n}^{(1)} = g\tau\,,\quad \bar{n}^{(2)} \sim n_o^{(2)}(E_z/E_{zc}^{(2)})^{6/4q-5} \tag{6}$$

The radius of the pinch, the maximum concentration, and the total current take the form

$$a_q^{(1)} \sim R(E_z/E_{zc}^{(1)})^{3/[2(2q-1)]}\,,\quad n_m^{(1)} \sim \bar{n}^{(1)}(E_z/E_{zc}^{(1)})^{3/(1-2q)}\,,$$

$$I^{(1)} \sim \pi R^2 e\,\bar{n}^{(1)}(\bar{\mu}_e+\bar{\mu}_h)E_{zc}^{(1)}(E_z/E_{zc}^{(1)})^{1/(1-2q)}\,, \tag{7}$$

$$a_q^{(2)} \sim R(E_z/E_{zc}^{(2)})^{6/(4q-5)}\,,\quad n_m^{(2)} \sim n_o^{(2)}(E_z/E_{zc}^{(2)})^{6/(5-4q)}\,,$$

$$I^{(2)} \sim \pi R^2 e n_o^{(2)}(\bar{\mu}_{eo}+\bar{\mu}_{ho})E_{zc}^{(2)}(E_z/E_{zc}^{(2)})^{1/(4q-5)}. \tag{8}$$

From (5), (7) and (9) it is readily seen that $q \neq 1$, 1/2, 5/4. From formulas (7) and (8) we see that the field $E_{zc}^{(1,2)}$ is the critical field at which $a_q = R$. The results are valid if it is possible to introduce a pinch radius, i.e., when $a_q < R$. This condition imposes limitations on the field: in the case of linear recombination, for we have $E_z < E_{zc}^{(1)}$ for $q > 1/2$, and $E_z > E_{zc}^{(1)}$ for negative q or $q < 1/2$.

Thus, depending on the scattering and recombination mechanism, we can obtain different dependences of the total current on the applied field. In the case of scattering by acoustic phonons, $q = -1/2$, we have $I_{-1/2}^{(1)} \sim E_z^{1/2}$ and $I_{-1/2}^{(2)} \sim E_z^{-1/7}$. In the case of scattering of electrons by holes or scattering of holes by phonons (the hole gas is non-degenerate), $q = 0$, we get $I_0^{(1)} \sim E_z$, and $I_0^{(2)} \sim E_z^{-1/5}$ (this result was obtained earlier in [10]). In the presence of quadratic recombination, we obtain a decreasing characteristic when $E_z > E_{zc}^{(2)}$ ($E_{zc}^{(1,2)} \sim 10$ V/cm at $T \sim 100$°K, $R \sim 10^{-1}$ cm, and $n_0 \sim 10^{16}$ cm^{-3}). A really decreasing characteristic should apparently signify instability of the current.

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