QUARKS WITH COMPLEX SPIN

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It was shown earlier [1] that it is possible to construct a formalism for a quantum description of "particles" with complex spin, in which such an individual "particle" does not have its own wave function, and is therefore unobservable, whereas a system of two "particles" with complex spins s_1 and s_2 can be described by a wave function if the difference $2s_1 - 2s_2$ is an integer. The total spin of the system assumes in this case either integer or half-integer values, and therefore states with definite total angular momentum can be interpreted as real particles. This makes it possible to consider a composite model, in which the elementary particles are assumed to be "diquarks," the quark assigned a complex spin 1).

In [1] we formulated the principles of the kinematics of "particles" with complex spins; in this paper we deal with their (non-relativistic) dynamics. We consider the case when the spins of both quarks are equal to -1/2 (this case, in which the diquarks have integer spin, is singled out by group-theoretical considerations. The wave function of a system of two "particles: with spin -1/2 in the c.m.s. can be realized as a function $\Psi(\vec{r}, \vec{a})$, where $\vec{r}$ is the relative radius vector and $\vec{a}$ is a spin variable common to both quarks (unit vector). Upon rotation, the wave function is transformed in accordance with

$$T_R\,\Psi(\mathbf{r}, \mathbf{a}) = \Psi(R^{-1}\mathbf{r}, R^{-1}\mathbf{a}). \tag{1}$$

The scalar product in the state space is of the form

$$(\Psi_1, \Psi_2) = \int da\,dr\,\overline{\Psi_1(\mathbf{r}, \mathbf{a})}\,\Psi_2(\mathbf{r}, \mathbf{a}), \tag{2}$$

and the spin operator towards act in accordance with the formulas

$$\mathbf{s}_1 = 1/2\left[-i\mathbf{a}\times\frac{\partial}{\partial\mathbf{a}} + i\mathbf{a}\times\left(\mathbf{a}\times\frac{\partial}{\partial\mathbf{a}}\right) + \mathbf{a}\right], \tag{3a}$$

$$\mathbf{s}_2 = 1/2\left[-i\mathbf{a}\times\frac{\partial}{\partial\mathbf{a}} - i\mathbf{a}\times\left(\mathbf{a}\times\frac{\partial}{\partial\mathbf{a}}\right) - \mathbf{a}\right]. \tag{3b}$$

The quark commutation operator is

$$P_{12}\,\Psi(\mathbf{r}, \mathbf{a}) = \Psi(-\mathbf{r}, -\mathbf{a}). \tag{4}$$

The most general form of the Hamiltonian of a two-quarks system in the c.m.s. is

$$H = \mathbf{p}^2/2m + U, \tag{5}$$

where U is a scalar function of the vectors $\vec{r}$, $\vec{s}_1$, and $\vec{s}_2$ at our disposal. In place of $\vec{s}_1$

1) The work "quarks" denotes here fundamental particles, without identifying them with the triplet of particles introduced by Gell-Mann and Ne'eman.

and $\vec{s}_2$ it is convenient to use as the arguments of U the Hermitian operators a and $S = -i\vec{a} \times \partial\vec{a}$ ($\vec{s}_1$ and $\vec{s}_2$ are Hermitian conjugates):

$$\mathbf{S} = \mathbf{s}_1 + \mathbf{s}_2, \tag{6a}$$

$$\mathbf{a} = (\mathbf{S}^2)^{-1}(\mathbf{s}_2 - \mathbf{s}_1 + 2\mathbf{s}_1 \times \mathbf{s}_2). \tag{6b}$$

The existence of the vector $\vec{a}$ is a specific property of a complex spin (in the case of integer or half-integer spins there is no Hermitian vector operator with commuting components).

Thus, $U = U(\vec{r}, \vec{a}, \vec{S})$. If we represent U in the form $U = \vec{S}^2/2I + V(\vec{r}, \vec{a}, \vec{S})$, where I is a constant with dimension of the moment of inertia, then the Hamiltonian (5) becomes identical with the Hamiltonian of a rigid rotator with moment of inertia I, located in an external field V which is spherically symmetrical (but depends on the angular velocity $\vec{\omega} = \vec{S}/I$). We note that the analogy with the rotator is formal, since the meanings of the variables $\vec{r}$ and $\vec{a}$ are entirely different.

Depending on the choice of the potential U, the Hamiltonian (5) can lead to a great variety of energy spectra. Thus, the spectrum may turn out to be rotational (without artificially introducing degrees of freedom corresponding to the rigid body). In spite of the existence of a centrifugal barrier, the energy may turn out to be independent of the total angular momentum J (spin of the composite particle). Such a spectrum arises if the interaction does not depend on the spins ($U = U(r)$). In this case the ground state for any value of J has a zero orbital angular momentum and a total spin $S = J$; the energy of this state does not depend on S, meaning also on J. For other choices, the energy may decrease or oscillate with increasing J.

The distinguishing features of the model appear also in the properties of the composite particles. Thus, for example, in the case $U = U(r)$, all the multipole moments turn out to be equal to zero for any value of J, for states with the lowest energy at the given J. In the case of quarks with spin 1/2, such a situation is impossible. The magnetic moment μ of the composite particle may turn out to be arbitrarily large, because the total spin of the two-quark system assumes arbitrarily large values for any fixed total angular momentum J. In the case of quarks with spin 1/2, the range of possible values of μ at a given J is greatly limited.

Thus, in spite of the fact that individual quarks with complex spin are not observable, it is possible to construct a quantum dynamics of a system consisting of two such quarks, namely, by introducing a Hilbert space of states and a Hamiltonian, and by taking into account the identity of the quarks. In the composite model based on this dynamics, the mass spectrum and the properties may turn out to be essentially different from those in the ordinary composite model, owing to the presence of a continuous spin variable. The composite model in which quarks are ascribed a complex spin is equivalent to a certain parametrization, in a definite group-theoretical sense, of the internal state of the particles. The model was apparently a heuristic value, analogous to a certain degree to the meaning of the shell model in the theory of the nucleus or the concept of quasiparticles in the theory of Fermi liquids.

[1] V. I. Roginskii, ZhETF Pis. Red. 8, 437 (1968) [JETP Lett. 8, 269 (1968)]; ITEF Preprint No. 647, 1968.

PHOTOPRODUCTION OF π^+ MESONS BY LINEARLY POLARIZED PHOTONS

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The need for additional measurements of the photoproduction of π^+ mesons by polarized photons has been discussed many times in both theoretical and experimental papers [1 - 4].

A polarized photon beam was obtained with the "Sirius" synchroton from interactions of electrons with energy E_e = 850 MeV ± 3.5% with a crystalline diamond target of thickness 0.0157 radiation units. The characteristics of the beam were determined with a magnetic pair spectrometer [5] of resolution $\Delta E_\gamma/E_\gamma$ = ± 3%. Good agreement was obtained between the theoretical and experimental spectra. The polarization at the 268-MeV peak of the coherent bremsstrahlung was calculated theoretically and amounted to 14.4%.

The figure shows the experimental setup consisting of scintillation counters C_1 (100 x 100 x 5 mm), C_2 (170 x 170 x 10 mm), C_3 (240 x 240 x 20 mm), C_4 (150 x 150 x 5 mm), n-detector (100 dia x 200 mm) made of plastic scintillators, and spark chambers SC_1 and SC_2. The chamber SC_1 has 0.2 mm aluminum windows and a 100 mm gap, and SC_2 is an eleven-gap chamber with 10 copper plates up to 2 mm thick. The signals from C_1, C_2, and the n-detector (neutron counter) were fed to a fast triple-coincidence circuit with resolution time 1.7 nsec. The signal from the triple-coincidence circuit, when not blocked by the counters C_3 and C_4, was fed to a scaler circuit and to the triggering circuit of the high-voltage genera-