FINE STRUCTURE OF GIANT RESONANCES

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The energy dependence of the cross sections of nuclear reactions contains resonances (called giant resonances), whose width greatly exceeds the average level width of the compound nucleus at the given excitation energy. In experiments with good energy resolution, the giant resonance is manifest as an aggregate of individual narrow peaks corresponding either to isolated levels of the compound nucleus, or to the Ericson fluctuation structure if the levels overlap. The analog resonances are examples that have been thoroughly investigated in this respect.

To explain the fine structure of giant resonance, it has been customary so far to use the model of the so-called input states [1,2], according to which the process of formation of the compound nucleus proceeds through several stages corresponding to motion up the ladder of the "state hierarchy" from simple particle-hole structures to more complicated collective excitations. The existence of a fine structure of the giant resonances is frequently regarded as proof of the validity of this hypothesis.

We show in this paper that the occurrence of the fine structure of the giant resonances is due to the unitarity relation. The use of the latter makes it possible to obtain both the energy dependence of the reaction cross section and the connection between the total and partial widths of the giant resonance, which differs from the usual one (the total width is not equal to the sum of the partial widths).

Assume that there is one broad resonance at a complex energy $E_0 - i\Gamma_0/2$ and an aggregate of narrow isolated levels $E_c - i\Gamma_c/2$, with

$$|E_c - E_0| \lesssim \frac{\Gamma_0}{2}, \quad \Gamma_c << \Gamma_0 \tag{1}$$

The spins of all the levels will be assumed to be the same. Assuming for simplicity that the initial (α) and final (β) particles are spinless, we write the partial amplitude of the reaction $M_{\alpha\beta}$ in the form

$$M_{\alpha\beta} = M_{\alpha\beta}^{(0)} + \sum_c M_{\alpha\beta}^{(c)}, \tag{2}$$

where

$$M_{\alpha\beta}^{(i)} = -\frac{1}{4\pi}\frac{M_{\alpha i}\; M_{\beta i}}{E - E_i + i\Gamma_i/2}, \quad i = 0, c \tag{3}$$

and $M_{\xi i}$ are the amplitudes of production of the intermediate state j in the channel $\xi = \alpha, \beta$. These amplitudes are connected with the partial widths $\Gamma_{\xi i}$ by the equations

$$M_{\xi i} = 2\pi i^{l} \phi_{\xi i}\left(\frac{\Gamma_{\xi i}}{k_\xi m_\xi}\right)^{1/2}. \tag{4}$$

in which m_ξ and k_ξ are the reduced mass and the wave number in the c.m.s. of channel ξ, and $\phi_{\xi j}$ is the real phase.

All the subsequent deductions are based on the fact that the second equation in (1) allows us to write down an approximate unitarity relation that establishes a connection between $M^{(0)}_{\alpha\beta}$ and the amplitudes $M_{\xi c}$ of the production of narrow states of the compound nucleus. The occurrence of this approximate relation can be explained as follows.

The subdivision of resonances into narrow and broad ones is equivalent to a classfication of the processes by their time duration (sharper energy dependences correspond to faster reactions). The amplitude $M_{\xi c}$ describes, in this sense, a fast process, inasmuch as it remains near E_c a smoother function of the energy than the resonance factor $(E - E_c + i\Gamma_c/2)^{-1}$. The set of amplitudes $M_{\xi c}$ and $M^{(0)}_{\alpha\beta}$ constitutes a matrix $\tilde{S}$ describing the "fast" processes (it is significant that the $\tilde{S}$-matrix contains no amplitudes $M^{(c)}_{\alpha\beta}$ corresponding to narrow resonances). The $\tilde{S}$-matrix defined in this manner should be unitary to the extent that the narrow level of the compound nucleus can be regarded as stable compared with the broad one. This statement can be explained as follows. If the collision of the particles produces in the channel α a long-lives state c of the compound nucleus, then the latter will leave the bounded region of observation before it has time to decay into the particles β. Such an interaction will be read by the instrument not as the reaction $\alpha \to \beta$, but as the production of the particle c. Another possible event is the production of a short-lived state 0, which decays within the given region. This event will, obviously be read by the instrument as the reaction $\alpha \to \beta$. The unitarity of the $\tilde{S}$-matrix denotes, in particular, that the sum of the probabilities of the indicated events equals (more accurately, nearly equals) unity.

A consequence of the unitarity of the $\tilde{S}$-matrix is, above all, the fact that the amplitudes $M_{\xi c}$, as functions of E_c, should have poles at the point $E_0 - i\Gamma_0/2$. Assuming that all ξ channels are two-particle, we obtain the following formulas

$$M_{\xi c} = 2\pi\gamma l^{-in_\xi\pi}\left(\frac{\Gamma_{\xi 0}}{k_\xi m_\xi}\right)^{1/2}\frac{\Gamma_0/2}{E_c - E_0 + i\Gamma_0/2}, \tag{5}$$

$$\Gamma_{\xi c} = \gamma^2\Gamma_{\xi 0}\frac{\Gamma_0^2/4}{(E_c - E_0)^2 + \Gamma_0^2/4} \tag{6}$$

In formulas (5) and (6) γ is a real constant or a slowly-varying function, and n_ξ is an integer.

The partial widths $\Gamma_{\xi 0}$ of a broad resonance are connected with its total width Γ_0 by the expression

$$\Gamma_0 - \sum_\xi \Gamma_{\xi 0} \equiv \Gamma_{sp} = \pi/2\gamma^2\rho_c\left(\sum_\xi \Gamma_{\xi 0}\right)^2 \tag{7}$$

Here ρ_c is the level density near E_0. Formula (7) is obtained from the unitarity relation for $\operatorname{Im} M^{(0)}_{\alpha\beta}$, which acquires, after averaging over the energy interval ΔE satisfying the inequality

$$\Gamma_c << \Delta E << \Gamma_0, \tag{8}$$

the form

$$\mathrm{Im}M^{(0)}_{\alpha\beta} = \frac{1}{2\pi}\sum_{\xi} k_{\xi} m_{\xi} M^{(0)}_{\alpha\xi} M^{(0)*}_{\beta\xi} + \frac{1}{4}\rho_c M_{\alpha c} M^{*}_{\beta c}. \tag{9}$$

Expression (7) is obtained from (9) by substituting in the latter formulas (5) and (3), subject to the additional condition

$$\gamma^2 \rho_c \Gamma_0 << 1, \tag{10}$$

which is equivalent (recognizing that $\gamma^2 \sim \Gamma_c/\Gamma_0$) to the requirement that the narrow levels be isolated:

$$\Gamma_c \rho_c << 1. \tag{11}$$

Formulas (6) and (7) can be verified experimentally (in particular, using giant analog resonances as an example; we note that a formula similar to (7) was obtained for this case in [3] by another method).

The foregoing analysis is not connected with any dynamic model. It shows that the physical cause of the appearance of the fine structure of giant resonance is essentially the resonant rescattering in the initial and final states.

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[1] H. Feshbach, Congres International de Physique Nucleaire, Paris, Juillet, 1, 227 (1964).
[2] C. Bloch, ibid. 1, 227 (1964) [sic!].
[3] D. Robson, Phys. Rev. 137, B535 (1965).

OSCILLATIONS OF ABSORPTION AND AMPLIFICATION OF ULTRASOUND IN INELASTIC SCATTERING OF ELECTRONS

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Singularities of galvanomagnetic phenomena in strong electric fields, in the case of inelastic scattering of electrons with emission of optical phonons, were predicted in [1].

The same scattering mechanism can lead to oscillations of absorption and amplification of sound in a many-valley semiconductor as a function of the sound frequency ω or the electric field intensity E.

Let us consider a semiconductor with two valleys that shift in opposite directions upon passage of a sound wave. The electric field E and the sound wave vector q are parallel and are directed symetrically with respect to the valleys. The numerical factors governed by the geometry of the valleys will be written out for the case of n-Ge with sound propagating along a fourfold axis. We choose as the sole scattering mechanism the emission by the electron of an intervalley phonon with energy $\hbar\omega_0$ at the instant when this is allowed by the energy conservation law. During all the remaining time the electron moves, without scattering, under the influence of the electric field and the deformation potential, along a straight